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Automated fixture design for a rapid machining process

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Abstract

Rapid prototyping techniques for CNC machining have been developed in an effort to produce functional prototypes in appropriate materials. One of the major challenges for rapid machining is to develop an automatic fixturing system for securing the part during the machining process. The method proposed in this paper is the use of sacrificial fixturing, similar to the support structures in existing rapid processes like Stereolithography. During the machining process, sacrificial supports emerge incrementally and, at the end of the process, are the only entities connecting the part to the stock material. This paper presents methodologies for the design of sacrificial support structures for a rapid machining process and illustrates them using a complex sample part machined in the laboratory.

Keywords: Rapid Prototyping, CNC Machining, Fixturing, Manufacturing

1. Introduction

Rapid prototyping (RP) processes automatically create physical prototypes from three-dimensional (3-D) CAD models in a short period of time (Pham and Gault 1998). The processes are intended for creating prototypes or very small batches of parts. Most commercial RP systems are based on additive processes (Upcraft and Fletcher 2003) whereby models are constructed by stacking 2⁵/₂-D cross sectional layers on top of one another. The additive RP systems are often limited in both geometric accuracy and material quality. Subtractive processes such as CNC machining have advantages over the limited choice of materials and the limited
functionality of parts produced by additive processes. However, machining is not a completely automated method in either the process or fixture planning steps. There has been a need for a rapid machining system, but previous attempts to automate CNC machining have been approached from the perspective of traditional machining methods. It has become necessary to re-think how parts can be held, oriented, and then actually cut; perhaps borrowing methods from existing approaches to additive rapid prototyping processes.

A new automated rapid machining method is being developed that is capable of creating low volume or prototype parts in a variety of materials (Frank, Wysk, and Joshi 2004). CNC Rapid Prototyping (CNC-RP) combines CNC machining with RP methodologies to create functional three-dimensional parts in a completely automated fashion. Since a conventional 3-axis CNC machine can only machine from one direction (Z axis), a rotary device (fourth axis) is required to rotate a part to enable the cutting tool to approach the part geometries from various orientations. In order to create a three-dimensional part using CNC-RP, the stock is oriented and machined about one axis of rotation until all necessary surfaces are machined. Rotating the stock about an axis would be a very difficult task if a conventional fixturing technique using vises, clamps, and/or v-blocks were used to hold the part during the machining process. Multiple manual set ups would preclude such an approach from being considered a viable rapid machining technique.

In some current additive RP processes, the “fixtures” used to secure the part are called sacrificial support structures, which are automatically added during the process. These structures support and increase the stiffness of overhanging features that do not have a preceding layer to support them from below. These sacrificial supports are then removed in a post processing step. CNC-RP proposes a similar
concept of sacrificial supports; however, instead of adding material to the physical model, the supports are added to the CAD model prior to tool path planning and subsequently created during the machining process along with other part features. The supports are currently implemented as small features added to the solid model geometry parallel to the axis of rotation. They are used as fixtures not only to provide stiffness to the part, but also to preserve the location information of the part during the machining process. There are two types of sacrificial supports, **permanent** and **temporary** supports, depending on their life cycle during the rapid machining process. Since the part can be held at a minimum with one support on each end, one permanent support is created on each end of the part during the machining process and, at the completion of the process, are the only entities connecting the part to the stock material. Temporary supports are also created during the machining process, but are subsequently removed by the end of the machining process.

Figure 1a illustrates the machining setup with one axis of rotation used in the CNC RP process. The approach uses an incremental machining strategy whereby the part is machined section by section in order to maintain stiffness of the part and support system during cutting. In this section-by-section method, the part and sacrificial supports are divided into at least 3 sections; 1) the part section, 2) the left support section and 3) the right support section (Figure 1b). The part section is machined first, which is illustrated in Figure 1c (steps 1-4). Since the supports are not yet created, the part section is supported by the remaining stock material in a very rigid setup. Next, the left and right support sections are machined, as illustrated in Figure 1d (steps 5-8). During each section’s machining, the stock is rotated several times about the axis of rotation until all surfaces in that section are completely machined. In Figure 1e (steps 9-10), the temporary supports are removed leaving the
finished part secured to the stock material by only the permanent supports. Lastly, the part is severed from the stock by machining or sawing through the permanent supports (step 11). The Steel part pictured below step 11 is a bicycle suspension component and was created in the lab using CNC-RP. From a CAD model, all process and fixture plans including NC code were generated in under one hour. It was machined in approximately 20 hours using one 3/16” end mill on a 3-axis Fadal VM15 with a 4th axis indexer.

Figure 1 - Rapid machining; (a) set up, (b) sections machining approach, (c) Part Section machining steps, (d) Support Section machining steps, and (e) Support removal steps
To date, we have developed a visibility method that can determine if an axis of rotation is feasible, and determine the minimum set of orientations such that all surfaces are visible. Next, a machinability method was developed that can determine if a tool can contact all the visible surfaces for each orientation (Frank, Wysk, Joshi 2006; Li and Frank 2006). The next major problem for a completely automated CNC RP process is automated fixturing. This paper presents an approach to solve the complex problem of automatically determining what sacrificial support fixturing schema will work for a particular part. The challenge is that this process must work for any CAD geometry sent to the system since a typical RP system assumes that the user has limited or no time/skill to create a set of process plans. In other words, the current RP systems are assumed to be nearly “push-button” machines. The approach presented attempts to simplify this nearly intractable problem using creative methods exploited in the RP technologies of the past two decades. This paper will cover 3 major areas necessary for sacrificial support development, 1) design, 2) machining sequence, and 3) support removal. The implementation section illustrates the proposed method by creating a sacrificial support fixture design for a complex example part.

2. Related Work

Traditional fixturing techniques use a number of workholding elements such as vises, clamps, V-blocks, modular plates, etc. These fixturing approaches require a great deal of skill and lack the flexibility to handle arbitrary part shapes easily. Some existing methods such as dedicated, modular and phase-change fixturing are more suitable for large batches or mass production, where the investment for set up time and fixture costs can be absorbed; however, not for rapid prototyping. There has been
some research dedicated to either fully subtractive or hybrid (additive/subtractive) RP systems and each has had to confront the problems related to fixturing arbitrarily shaped parts. Merz et al. (1994) presented Shape deposition manufacturing (SDM) as a hybrid approach using both additive and subtractive processes. The models are constructed in a layer-based manner through sequential deposition and machining steps. Support materials are added depending on whether or not the layer contains undercut features.

Several researches have focused on developing fixturing systems for RP processes. Sarma and Wright (1997) and Choi et al. (2001) presented a process called Reference Free Part Encapsulation (RFPE) that uses a low melting point material to encapsulate the stock during machining. Shin et al. (2003) presented High-speed Rapid Prototyping (HisRP) as a process that combined high-speed machining with an RFPE process. This fixturing system provides a rigid support structure for resisting cutting forces during the machining process and can accommodate any arbitrarily shaped workpiece; however, the process introduces thermal shrink and expansion problems that can limit the ability to create accurate parts. Hazony and Zeidner (1994) presented the application of SDTM (Seamless design-to-manufacture) technology to RP. Their method was applied to the fabrication of marine propulsers. A structure support beam was used as a fixture to hold the propulsers during the machining process. However, the authors did not describe the analysis of the support beam and the applicability of using the approach with parts other than marine propulsers.

There has been an interest in creating rapid machining systems that can create automatic fixtures. Lennings (2000) discussed using CNC machining as an RP process and there is existing software that attempts to implement fixturing for rapid machining. Millit and DeskProto are commercial software packages for generating
numerical control (NC) code from STL (Stereolithography) files. In the Millit
process, the software decomposes a model into several thick slabs, where each slab is
called a component. The fixturing system consists of outer frames and bridges (thin
strips) that connect the components to the frames and act as fixtures during machining.
The overall RP method is not an automated system since it requires a significant post-
assembly process whereby the finished part needs to be bolted, glued, welded, or
otherwise bonded to create a complete part. DeskProto uses a similar fixture
approach, with bridges added to a model for fixturing during the machining process.
The software uses only four available bridges (left, right, front and back) for every
part. There is a more recent version for machining a part in several orientations called
N-sided milling which uses a rotary axis and bridge supports. Unfortunately, the
software does not include bridge design or analysis to determine if a feasible solution
can be developed for fixturing an arbitrary part.

Although there has been significant interest in creating a rapid machining
system that has automated fixturing, no suitable approach has been found in the
current literature. Herein, we provide evidence that our proposed approach has shown
promise of a viable solution. This paper provides a comprehensive solution approach
that will clearly show that sacrificial fixturing can be used in a subtractive process in
an automated fixture design system.

3. Overview of the Sacrificial Support approach

The following section provides an overview of the goals in creating a support
structure design and a general approach as to how the design will be derived from
only the simple geometry of an STL model. The application challenges in this work
are centered on the basic assumption that an RP system does not require skill or effort
on the part of the operator. The technical challenges in developing this system are the following: 1) deflection of the part needs to be minimized, 2) machinable surface area of the part needs to be maximized, and 3) the system must be flexible enough to handle arbitrary part shapes.

It is assumed that the axis of rotation is already determined before starting the design process. It is also assumed that, sacrificial supports are only used on the ends of the part, attaching to the existing stock material, and all supports are parallel to the axis of rotation. As such, the sacrificial supports in our system are simple extruded features added to the ends of the CAD model of the part. These new features are considered new surfaces on the part, at least temporarily, and therefore, a process plan to machine the part will include toolpaths to create these new features along with the rest of the original part.

![Figure 2 - Design parameters](image)

Similar to other RP processes, a sliced STL model is the input for the support design process. The design parameters that need to be considered are the length, shape, size, number, and location of sacrificial supports (Figure 2). The part properties, such as size and length of the part, can influence toolpath planning as well as the support design parameters. For example, in order to machine a large diameter part, a longer cutting tool is required to access part surfaces. The diameter of a tool
generally increases as the length requirement of the tool increases (based on currently available tools used in machining). Consequently, the shortest possible length of a support will be limited by the size of the tool used in the support section. In other words, if we need to machine completely around the ends of the part, then there must be at least enough room to fit the tool diameter between the end of the part and the remaining stock material (Figure 2). Thus, before any sacrificial support system is designed, we know that at least one support on each end will at least be some minimum length. As mentioned previously, we designate these supports, the **permanent** supports. The actual length of the **temporary** supports will then depend on the final layout (about the ends of the part) and the size (diameter) of the part and the supports.

The ultimate goal of a sacrificial fixturing system layout is to produce an accurate part, which will require a robust design methodology, given only an STL model of a part. Unlike most RP systems, a subtractive process applies considerable forces to the part material, therefore the structural characteristics are critical. However, it is equally important that the sacrificial support layout allows the cutting tool to access surfaces of the part effectively. For a larger volume production part, the fixture design process would entail considerable analysis and design iterations, using advanced simulation and Finite Element Modeling. For a rapid manufactured part, this is not an option, and therefore, the following sections describe a non-traditional design process that is motivated by the needs of a truly push-button system. In the next section, we describe the set of design parameters for a sacrificial support structure, how they affect the successful creation of a part and our current methodology for solving this design challenge.
4. Design parameters

In the current method of fixture design, a small set of design parameters has been chosen. These parameters are based on an assumption that supports are simple extruded features that extend from the ends of the part to the stock. The design parameters include the number of, length, size (diameter), shape (cross section) and location of the supports. Each parameter is presented with regard to its effect on part quality and processing capability and then the current decision approach is presented.

4.1 Length of the sacrificial supports

Since the supports in our system are an order of magnitude smaller (in cross section) than the part itself, we assume that the deflection of the part will be a result of the deflection of the small supports attaching it to the stock. It is obvious that the length of a support should be minimized in general in order to minimize part deflection.

During the machining process, two general types of deflection can occur, bending and torsion. One can compare these deflections by using mechanics of materials theory. Figure 3 illustrates the deflection of a cylindrical beam due to bending and torsion.

![Figure 3](image-url)

Figure 3 - Beam deflection due to (a) bending and (b) torsion
Since our part/fixture system is fixed at both ends to the remaining stock material, we can use a statically indeterminate fixed-end beam model. From theory of mechanics of materials (Hearn 1997; Salter 2000) the maximum deflection \( y_b \) due to bending of a beam with fixed ends is

\[
y_b = \frac{FL^3}{192EI}
\]

(1)

where \( F \) is the cutting force, \( L \) is the length of the beam and \( E \) is Young modulus. \( I \) is the second moment of inertia of the beam which is equal to \( \frac{\pi}{64}D^4 \) (\( D \) is the diameter of the cylinder). For torsion, the maximum deflection is a function of the angle of twist. The maximum angle of twist \( (\theta_t) \) with the fixed ends is

\[
\theta_t = \frac{TL}{4JG}
\]

(2)

where \( T \) is a torque from the cutting force, \( J \) is the polar moment of inertia which is \( \frac{\pi}{32}D^4 \) for a cylinder, and \( G \) is the Shear modulus. The deflection from torsion for a beam of radius \( R \) can be measured by the distance that a point on the beam is deflected by the cutting force, which is:

\[
R\sin\theta_t = R\sin\left(\frac{TL}{4JG}\right)
\]

(3)

Designing the supports accurately and consistently with respect to both bending and torsion would be difficult, if one considers all possible machining conditions. To simplify, the sacrificial support design in this paper is based on the major deflection source, torsion (angle of twist). Figure 4a illustrates the deflection from torsion and bending when the radius of a cylinder is held constant and the length is varied. The deflection from torsion is higher than that from bending when the \( L/R \)
(Length/Radius) ratio is between about 0 to 8. The current approach is to choose 4 for the \( L/R \) ratio as the maximum point that we can consider torsion as the major deflection since torsion is about 80% of total deflection (Figure 4b) and total deflection is still quite small. Thus, for a support that has \( L/R \) ratio equal or less than 4, we assume the major deflection source is torsion.

![Graphs showing beam deflection due to bending and torsion](image)

**Figure 4 – Beam deflection due to bending and torsion**

In the previous section, we indicated that the minimum length of a support would be limited to being as small in length as the diameter of the largest tool used to machine the end of the part. Table 1 illustrates likely tool sizes that can be used for each part size (diameter). It should be noted that the part diameter is measured as the minimum diameter of the stock material that can be used based on the axis of rotation for the part. The tool sizes represent commonly available diameters for extra-long milling tools in the prescribed length; however custom tools may be used in a rapid machining system. Support diameter selection will be presented in a later section of
this paper, but it is important to note that the current approach will not prescribe a support with \(L/R > 3\), which is below our maximum value that we assume the major deflection source is from torsion.

<table>
<thead>
<tr>
<th>Part diameter / Tool Length (in)</th>
<th>Tool diameter / Minimum support length (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\leq 2)</td>
<td>0.1250</td>
</tr>
<tr>
<td>(2 &lt; D_{part} \leq 4)</td>
<td>0.1875</td>
</tr>
<tr>
<td>(4 &lt; D_{part} \leq 6)</td>
<td>0.2500</td>
</tr>
<tr>
<td>(6 &lt; D_{part} \leq 8)</td>
<td>0.3750</td>
</tr>
<tr>
<td>(8 &lt; D_{part} \leq 10)</td>
<td>0.6250</td>
</tr>
</tbody>
</table>

Table 1 - Part diameter \(D_{part}\) and support length

Since there are two types of sacrificial supports, permanent and temporary, the requirement of the length for each type are prescribed differently. The permanent supports need to hold the part through the end of the process, so the \(L/R < 4\) ratio is enforced for them. In our approach, the initial design iteration for the support system prescribes a permanent support scheme (one on each end) that is able to satisfy the maximum deflection criteria (create an accurate part). The temporary support may be equal or longer than the permanent support since it is only added for increased stiffness to the system and hence a more conservative and robust design.

Although adding more supports can decrease the deflection, adding a temporary support that is too long and slender will not contribute much to the stiffness of the system. Moreover, each additional support blocks the accessibility of the tool to some part surfaces, at least temporarily. Thus, our current approach is to limit the length of temporary supports to an \(L/R\) ratio of 10. As noted in Figure 4a, deflection abruptly increases when \(L/R\) ratio is larger than 10, primarily due to bending.
4.2 Shape of sacrificial supports

Sacrificial supports could be created using any extruded cross-section such as circles, squares, ellipses, rectangles or some other polygonal shape. For the current implementation we are using cylindrical supports. Since a slice file is used to provide information about the part geometry, the area of each slice boundary represents the possible space for locating the supports on each end of the part. The slice boundary is offset inwardly equal to the radius of the intended supports. This offset boundary represents the polygon on which the center of the supports can be located (Figure 5).

![Figure 5 - Offset polygon](image)

Cylindrical sacrificial supports have been implemented on many sample parts in the laboratory, but there are some foreseen limitations. For example, a part that has an end that is similar to a thin plate will have a very slender cross section and therefore require a very small diameter support. An elliptical shaped support may be a more suitable shape in this case; however, locating an ellipse on the slice plane will require a different locating method. It will entail the additional geometric problem of fitting a shape with a variable orientation angle inside a polygon (an ellipse will fit on the end of a plate best in one particular orientation). Selecting an optimal support shape for each part’s geometry will be considered in future work. Except for some limitations with thin part shapes, a cylindrical shape has the advantage of easy placement on the end of any arbitrary shaped part and it is straightforward to approximate deflection without having to consider orientation as a variable.
4.3 Size of sacrificial support

As with the length of the support, the diameter greatly affects the stiffness of the system. Although increasing the size of the sacrificial support generally reduces the deflection, the machinable surface area of a part decreases with support size. This can be illustrated using a part with a spherical shape whereby an increasing size support is seen as a plane intersecting incrementally deeper into the sphere. The section that is cut from the sphere by the plane is the non-machinable surface area. Of course, this is also dependent on the geometry of the surface part where the support is attached.

The current approach to selecting a support size considers the worst-case scenario with respect to deflection. The idea is to use a set maximum deflection allowable for the part based on the part tolerance and then design the support as if there were only one permanent support used on each end and the supports were located in the worst location. Since we assume torsion is the major deflection source, the worst-case will be when the moment arm is maximized when the permanent support used on each end is located across the diameter of the part, opposite a tangentially applied cutting force. Finite Element Analysis (FEA) was used to verify the equivalent size of support required to maintain deflection below some prescribed value. Figure 6 illustrates the size of support necessary to keep deflection below 0.003 in (0.076 mm) for different part sizes. This amount of deflection was chosen arbitrarily and would vary based on the desired part tolerance. Although this seems to be a straightforward mechanics analysis, recall that the support lengths will change based on the tool required to machine around the ends of the part. For each diameter increase in part size, there is a related increase in minimum tool diameter. For this work, we restricted ourselves to commercially available tool sizes as a practical
approach, although in theory custom tools could be used that increase linearly in diameter versus length.

![Graph of Support Diameter vs Part Diameter]

Figure 6 - Support diameter

From the data illustrated in Figure 6, the required diameter of support can be estimated using linear regression where:

\[ D_{Support} = 0.0475D_{Part} + 0.0709 \]  

(4)

The size of the support will obviously be different if the maximum allowable deflection changes; however, this simply entails a change in the y-intercept of this linear equation, which can vary with the specified part tolerance. Recall, this approach prescribes the necessary size of each permanent support if only the two permanent supports are used, and they are placed in the worst location along the end of the part. Therefore, any improved location of the permanent support and any use of temporary supports will generally add a factor of safety to the worst-case deflection conditions. If temporary supports are prescribed, they too will use the diameter as calculated above. Although this does not provide an optimal support size, the goal in this rapid system is to guarantee a good part, first time, every time. The next section describes how the use of temporary supports is determined and how many supports are proposed for an effective sacrificial fixture layout.
4.4 Number of supports

In the proposed method, the part has to be held by at least one permanent support on each end, therefore any additional support is considered a temporary support. Although additional supports generally increase the stiffness of the system, increasing the number of supports introduces other design problems. For instance, more supports generally decreases the machinable surface and obstructs tool accessibility, which increases the number of required setup orientations (Figure 7).

![Accessibility of cutting tools from end view of the part: (a) two supports, (b) three supports and (c) four supports](image)

Our current approach is to use only two supports at most on each end, one permanent and one temporary, for a total of 4 sacrificial supports. This is reasonable since the permanent supports are already designed to satisfy the maximum deflection allowable and the temporary supports only make the structure stiffer. The approach is to attempt to add only one temporary support on each end, but only if it is both feasible and advantageous. The final number of support combinations can only be 1:1, 2:1 or 2:2 for each end (where \( n:n \) indicates the number of supports on each end of the part). In order to add the temporary support, the proposed method considers the accessibility of the tool between the supports. In equation 5, \( D \) is the diameter of the tool, \( D \) is the distance between the center of the two supports on an end, and \( r \) is the radius of the support. From Figure 8, the accessible angle (\( 2\theta \)) is:
\[ 2\theta = 2\cos^{-1}\left(\frac{D_r + 2r}{D}\right) \] (5)

Figure 8 - Tool accessibility

If \( D \) is equal to \( k(D_r + 2r) \), the accessible angle of the tool between the supports is:

\[ 2\theta = 2\cos^{-1}\left(\frac{1}{k}\right) \] (6)

When \( k \) increases, the accessibility of the tool increases. Figure 9 illustrates the change in angle as \( k \) varies (\( k \) is an arbitrary multiplier).

Although the tool can cut between supports placed at least \( D_r + 2r \) apart, the tool can only access the space from one direction (plus or minus 180 degrees). This will increase the number of orientations required in the machining process, since it is likely that this one particular angle was not otherwise necessary in the process plan.

Figure 9 - Accessible angle versus support spacing
Although the choice appears arbitrary, we note that the accessibility significantly increases from 0 to 120 degree when $k$ increases from 1 to 2 (Figure 9), and therefore the proposed method requires $D$ to be at least twice of $D_t+2r$. Since the largest tool diameter ($D_t$) used in support section is equal to the length of the permanent support ($L_p$) and support diameter is calculated by equation 4. Then, the distance between the centers of the supports ($D$) is at least:

$$D \geq 2(L_p + 0.0475D_{part} + 0.0709) \quad (7)$$

In summary, the approach to determining the number of supports is as follows; at most two supports are used if and only if the second (temporary) support can be located both 1) far enough from the permanent support and 2) not be too long and slender, as prescribed by the $L/R$ ratio (<10) discussed previously. This provides a stiffer support structure whenever possible, but avoids adding a second support when it is likely to reduce accessibility with little improved stiffness.

### 4.5 Location of sacrificial supports

In order to locate the support, the support size is initially calculated by equation 4. Then, the boundary of the polygon(s) on the slice file is/are offset inwardly equal to the radius of the support. The permanent support is located on the first slice from the end of the part where the support’s cross-section is completely contained within the slice polygon. The temporary support is located by searching through the slices from the end of the part to the slice that corresponds to the maximum length, as prescribed by the $L/R < 10$ specification. Hence, we are searching along the end of the part to find the best location to fit the temporary support within our design parameters. As mentioned previously, the supports must also be located far enough apart, or else only the permanent support will be employed.
Although the different layers constitute a 3D layout problem, our current approach is to treat the 2D slices as a projection of possible search areas. This is possible because we begin with a design that has two short permanent supports on each end that should satisfy part tolerance even if the temporary supports are not placed optimally.

Support layout designs in this research are intended to minimize the deflection due to torsion. From theory of mechanics of materials, the deflection of a fixed-end beam due to torsion depends on a torque that causes the twisting action. The torque is a function of the magnitude of the cutting force and the distance from the applied force to the centroid of the beam (moment arm). Obviously, the worst-case will be when the cutting force is applied with the longest moment arm. Thus, reducing the length of the moment arm generally decreases twisting. In the case where only the single permanent support is used on each end, the method is to minimize the moment arm of the cutting force. Figure 10a illustrates the moment arm ($M_A$) for the single support end. The objective function is quite simple:

$$MIN[M_A]$$

(8)

In order to locate the support, the length of the moment arm needs to be determined. The support section is approximated as a cylinder that can contain this section along the axis. While the support is located on the slice polygon, the location of the support should minimize the distance from the center of the support to the surface of this cylinder.

Figure 10 - Part end with (a) one support and (b) two supports
In the case of using both permanent and temporary supports, the distance between the supports is also important, in addition to the moment arm of the cutting force. As the supports are placed farther apart, the stiffness of the support section tends to increase. Figure 10b illustrates the moment arm \((M_A)\) and the distance between the supports \((D)\) for a 2-support end. Determining the length of the moment arm for a two-supported end is similar to the calculation for a single support. However, two supports may be located on different slice locations. Thus, the approximate cylinder will contain the slice polygons from the end of the part to the slice polygon where the longest support could be attached (a support with an \(L/R\) ratio of 10). Assuming the cutting force can be applied on any point along the cylinder surface, the moment arm will be the longest distance from the perimeter of the cylinder to the center of the imaginary line connecting the supports.

The objective function in this case is:

\[
MIN[\alpha M_A - \beta D]
\]  

(9)

In general, we assume that a good layout of supports involves supports that are located far apart, but are generally centered about the axis of rotation for the part in process.

In order to determine the coefficient \(\alpha\) and \(\beta\) in equation 9, the approach considers the deflection results of a 2-support layout from FEA and then uses multiple curvilinear regression methods (Barnes 1994; Hogg and Ledolter 1987; McCuen 1985). In this case, a second-degree polynomial equation with two independent variables is used. FEA is used to predict the deflection for 5 different levels of \(D\) and \(M_A\). Table 2 illustrates the FEA results for parts with diameter equal to or less than 1 inch. It should be noted that since we assume the deflection is restricted to the supports, the “part” is approximated as a cylinder.
The FEA results in Table 2 are based on aluminum; however, the results should be applicable regardless of the materials commonly used in this process. That is, we are simply searching for an equation describing a good layout condition. Then, we will use this equation to consider the possible locations of support from the locations along the polygons of the slice file. The FEA results are analyzed using multiple curvilinear regression method, as shown in Figure 11.

### Table 2 - FEA results with 5 different levels of D and MA (FEA Unit x10^-4)

<table>
<thead>
<tr>
<th>MA (in)</th>
<th>0.500</th>
<th>0.575</th>
<th>0.650</th>
<th>0.725</th>
<th>0.800</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (in)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.500</td>
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<td>0.896</td>
<td>1.082</td>
<td>1.305</td>
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<tr>
<td>0.575</td>
<td>0.589</td>
<td>0.701</td>
<td>0.838</td>
<td>1.001</td>
<td>1.269</td>
</tr>
<tr>
<td>0.600</td>
<td>0.567</td>
<td>0.659</td>
<td>0.789</td>
<td>0.964</td>
<td>1.183</td>
</tr>
</tbody>
</table>

### Regression Analysis:

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.00004706</td>
<td>0.00003294</td>
<td>-1.43</td>
<td>0.169</td>
</tr>
<tr>
<td>MA</td>
<td>0.00017535</td>
<td>0.00006256</td>
<td>2.80</td>
<td>0.011</td>
</tr>
<tr>
<td>D</td>
<td>0.00022715</td>
<td>0.00005448</td>
<td>4.17</td>
<td>0.000</td>
</tr>
<tr>
<td>MA^2</td>
<td>0.00033702</td>
<td>0.00003296</td>
<td>10.23</td>
<td>0.000</td>
</tr>
<tr>
<td>MA*D</td>
<td>-0.00068373</td>
<td>0.00008272</td>
<td>-8.27</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 1.550995E-06   R-Sq = 99.7%   R-Sq(adj) = 99.7%

Figure 11 – Regression Analysis result

The objective function is as follows:

\[
MIN \left[(-0.47 + 1.75MA + 2.27D + 3.37MA^2 - 6.84MA*D)\times10^{-4}\right] \tag{10}
\]

Using the same approach, the objective functions for part diameters up to 10 inches are presented in Table 3.
The objective function will be chosen using the diameter of the part. The location of 2 supports on each end can be determined by minimizing the result of these equations through all possible combinations of locations. Although this approach is not elegant, it is much more accurate than to find a general objective function for all part sizes. In the next section, we present a method for sequencing the machining operations that will generally minimize the overall deflection during the machining process.

5. Machining sequence

Once the supports are designed and located on both ends of the part, determining what sequence the process plan should take is the next task. As described in the introduction to this paper, the entire process is broken into 3 sections, the part section and then the two support sections. These sections are actually regions along the axis of rotation were machining will occur. We know that the part section will always be machined first. Next, we must determine which end of the part will have its supports created first. Before generating the machining sequence, the

<table>
<thead>
<tr>
<th>Part diameter (in)</th>
<th>Objective functions ($10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 &lt; D_{part} \leq 2$</td>
<td>$MIN[0.37 + 2.14 M_A - 2.66 D - 1.48 M_A^* D + 1.91 D^2]$</td>
</tr>
<tr>
<td>$2 &lt; D_{part} \leq 3$</td>
<td>$MIN[0.31 + 1.50 M_A - 1.86 D - 0.75 M_A^* D + 0.95 D^2]$</td>
</tr>
<tr>
<td>$3 &lt; D_{part} \leq 4$</td>
<td>$MIN[0.12 + 0.72 M_A - 0.79 D - 0.27 M_A^* D + 0.31 D^2]$</td>
</tr>
<tr>
<td>$4 &lt; D_{part} \leq 5$</td>
<td>$MIN[0.26 + 0.70 M_A - 0.95 D - 0.22 M_A^* D + 0.30 D^2]$</td>
</tr>
<tr>
<td>$5 &lt; D_{part} \leq 6$</td>
<td>$MIN[0.34 + 0.12 M_A - 0.25 D - 0.04 M_A^* D + 0.01 M_A^2 + 0.05 D^2]$</td>
</tr>
<tr>
<td>$6 &lt; D_{part} \leq 7$</td>
<td>$MIN[0.18 + 0.45 M_A - 0.60 D - 0.10 M_A^* D + 0.13 D^2]$</td>
</tr>
<tr>
<td>$7 &lt; D_{part} \leq 8$</td>
<td>$MIN[0.29 + 0.17 M_A - 0.26 D - 0.02 M_A^* D - 0.01 M_A^2 + 0.03 D^2]$</td>
</tr>
<tr>
<td>$8 &lt; D_{part} \leq 9$</td>
<td>$MIN[0.13 + 0.36 M_A - 0.39 D - 0.06 M_A^* D + 0.07 D^2]$</td>
</tr>
<tr>
<td>$9 &lt; D_{part} \leq 10$</td>
<td>$MIN[0.47 + 0.12 M_A - 0.25 D - 0.02 M_A^* D + 0.03 D^2]$</td>
</tr>
</tbody>
</table>

Table 3 – Objective function of the part diameter up to 10 inches
maximum deflection of each section has to be determined. Unfortunately, theory of mechanics only solves the problem where the supports and part are concentric (and in a 1:1 support layout only). For non-concentric supports in our 1:1, 2:1, and 2:2 layouts, this approach is not feasible. However, if the supports on each end can be approximated as a concentric beam, the deflection on each end can be predicted by simple mechanics analysis. Thus, in the current method, the part and supports are approximated by a concentric statically indeterminate beam that has different diameter left and right end beams. A bigger beam on one end represents the *stronger* support structure while the smaller beam represents the *weaker* support structure (Figure 12). Refer also to Figure 1 for an example part showing the different sections.

![Concentric statically indeterminate beam](image)

Figure 12 – Concentric statically indeterminate beam

The part section is machined first to reduce the overall deflection of the part. This section is fixed to and supported by the stock material on each end. The stiffness of the system depends on the connecting area at the section boundary, which is always larger than the support attachments created later in the process. Since the worst-case for torsion occurs where the cutting force is applied with the longest moment arm, the part is approximated as a cylinder with a radius \( R \) calculated from the minimum radius cylinder that could contain the part. The torque \( T \) is applied in the middle of the part. Figure 13a illustrates the related parameters in this section. The twist angle in this section is:
\[ \theta_1 = \frac{TL}{2\pi R^3 G} \]  

(11)

The next section will be the support section on the left or right side. If the radius of the support is \( r_1 \) and the length of support is \( l_1 \) (Figure 13b), then the twist angle from machining this section is:

\[ \theta_2 = \frac{(2l_1)TL}{(l_1 R^4 + Lr_1^4)\pi G} \]  

(12)

The last section will be the remaining support end. Since all supports are created, the maximum twist angle will occur during the machining of this remaining section. If the radius of the support in this section is \( r_2 \) and the length of the support is \( l_2 \) (Figure 13c), then the twist angle from machining this section is:

\[ \theta_3 = \frac{(l_1 l_2 R^4 + l_2 Lr_1^4)2T}{(l_1 r_2^4 R^4 + Lr_1^4 r_2^4 + l_2 r_1^4 R^4)\pi G} \]  

(13)

\[ \theta_3 > \theta_2 > \theta_1, \quad \theta_{\text{max}} = \theta_3 \]  

(14)

From equations 11, 12, and 13, the twist angles depend on the length and radius of the part and supports. The length of each section can be simply specified by the section

Figure 13 - Sectional analysis for (a) part section, (b) right support section and (c) left support section
boundary. Unfortunately, transforming non-concentric supports to concentric supports is not straightforward. In order to simplify the analysis, we assume that the same diameter machining tool creates the supports on both ends. Then, the lengths of supports on each end is equal to the largest tool diameter \((D_t)\). Since the maximum twist angle occurs in the last section, the maximum twist angle of the last section becomes:

\[
\theta_{\text{max}} = \frac{(D_r^3R^4 + D_tLr_1^4)2T}{(D_pr_2^4R^4 + Lr_1^4r_2^4 + D_tLr_1^4R^4)\pi G}
\]

(15)

From equation 15, \(\theta_{\text{max}}\) depends on \(r_1\) which is the radius of the support machined first. If \(r_1\) is larger than the radius of the second support \((r_2)\), the twist angle in the last section is larger. Thus, in order to reduce twist in the last section, the smaller support (weaker support side) is machined before machining the larger support (stronger support side).

In a 2:1 support layout, the end that contains 2 supports is generally going to be more rigid than the single support end since the effective size of the support is greater. When a single support is used on both ends (1:1 support layout), the support located with a longer moment arm is assumed weaker than the support with a shorter moment arm. Lastly, in a 2:2 support layout the distance between the supports \((D)\) and the moment arm \((M_A)\) are used to determine the relative strength of the support ends, using equation 10 or the equations in Table 3 for calculations.

The machining sequence proved to be quite straightforward: always machine the weaker support side first. In this section we presented a very simple method for comparing the relative strength of the two ends and have shown that machining the weaker side first will generally reduce part deflection during machining.
6. Support removal

After the part and support sections are created, temporary support(s) will be removed in a final machining process. The task will be to completely remove the temporary support(s) and machine the remainder of the surfaces that are blocked by this/these support(s).

The setup orientations for removing the temporary supports will be the same as the orientations for machining the part surfaces as calculated from the visibility algorithms used in the rapid machining method (Frank, Wysk, and Joshi 2006; Li and Frank 2006). The machining boundary is created by offsetting the support geometry with the smallest tool diameter that was used in the part section (Figure 14).

The size of the tool used to remove the supports must be equal to or smaller than the smallest tool used for machining the part or support sections. The tool machines within this boundary to the furthest visible surface in each orientation.

The proposed method ensures that the temporary supports are completely removed and the remaining visible surfaces that were previously blocked by the support are machined. We know that all part surfaces will be machined in the last process steps since the process plans have already been designed to machine all surfaces. The only difference is that we begin by machining with the temporary
supports in place, then machine again after the temporary supports have been removed. This is accomplished easily by generating toolpaths in CAM twice; once with the supports present on the CAD model and then a second time after the supports are removed from the CAD model.

### 7. Implementation

In this section a complex part, a model of a human femur bone, is illustrated and the steps of the sacrificial support design process are presented. The design methodology was implemented in C++ and used in conjunction with the CNC RP process to create process and fixture plans for rapid machining. A part such as this would be difficult or impossible to handle with traditional fixturing approaches.

The bone is a scale model with a length of 7” (177.8mm) and width of 1.5” (38.1mm). From equation 4, the size of supports has been calculated to be 0.14” (3.556mm). The shortest support length (permanent support) is 0.125” (3.175mm) based on Table 1. The maximum length of the temporary supports is 0.7” (17.78mm). Subsequently, the design process begins by slicing the STL model at 0.01” (0.254mm) slice spacing. The slice boundary is offset inward equal to the support radius. The first slice that can contain a support is used to locate the permanent support on each end.

![Figure 15 - Section boundary; (a) part section and (b) support sections](image-url)
The part section boundary is specified as the ends of the part. Therefore, most part surfaces are created in this section. The support sections are defined by the furthest slice from the ends of the part where the longest temporary support could be located. Figure 15a and 15b illustrates how the part is divided into three sections: 1) part section, 2) left support, and 3) right support.

Each support section is approximated as a cylinder as illustrated in Figure 16. The location of the supports on each end is determined by minimizing the objective functions (Table 3), considering the distance between supports and the moment arm. Table 4 illustrates support location results and their deflection estimates for the example femur model.
According to Table 4, the right end is relatively weaker (larger deflection estimate) therefore the machining sequences for the bone are 1) part section, 2) right support, and then 3) left support. After attaching sacrificial supports to the CAD model, the machining toolpaths are generated based on the existing methods for CNC-RP. Three orientations about the axis of rotation were required for this example part. The femur bone model, machined in Delrin plastic, is illustrated in Figure 17. This part was completed in approximately 10 hours using a 1/8” endmill on a 3-axis Fadal VM15 with a 4th axis rotary indexer. The part was machined from 3 orientations, with a layer depth of 0.003” (0.0762mm) and an approximate feedrate and speed of 150ipm and 5000rpm, respectively.

Table 4 - Location of supports and deflection estimate

<table>
<thead>
<tr>
<th>Support</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Deflection Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right end:</td>
<td>(inch)</td>
<td>(inch)</td>
<td>(inch)</td>
<td>(inch)</td>
</tr>
<tr>
<td>- permanent</td>
<td>-0.2132</td>
<td>-0.3069</td>
<td>0.02</td>
<td>0.1720*10^-4</td>
</tr>
<tr>
<td>- temporary</td>
<td>0.0795</td>
<td>0.5579</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Left end</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- permanent</td>
<td>0.0463</td>
<td>0.3667</td>
<td>6.73</td>
<td>0.0141*10^-4</td>
</tr>
<tr>
<td>- temporary</td>
<td>-0.0528</td>
<td>-0.4778</td>
<td>6.77</td>
<td></td>
</tr>
</tbody>
</table>

Figure 17 - Femur bone model; (a) with supports, (b) finished
8. Conclusion

This paper presents a method for creating sacrificial support fixtures in a rapid machining process. The approach has several advantages: 1) it has the capability of providing a completely automated rapid machining system, 2) it is exceedingly flexible in securing a vast array of complex parts, and 3) it requires little human intervention or skill to utilize. The approach to fixture design has been shown to work well for several complex parts machined in the laboratory. To date, we have created several parts in materials such as plastic, gray and ductile iron, aluminum and steel. Of all parts created so far, the worst dimensional variation measured was 0.005”, although at least half of that variation was accounted for by runout error measured in the indexer. Other parts have met or exceeded the part accuracy requirements we have set. Although we have always used sacrificial fixturing in the CNC-RP process, this work can be applicable in more traditional subtractive operations. For example, any complex part that has to be fixtured in multiple setups could benefit from a sacrificial support methodology like we have presented.

However, there are several opportunities for improvement. We recognize that the method is not always elegant; however, it is robust and has to date always provided feasible solutions that meet part requirements. The method is perhaps overly conservative, though, and could be improved with a more accurate beam model to provide more precise results for determining a machining sequence that yields better stiffness. Another improvement lies in considering the direction of cutting force in the support design, since the process planning algorithms for CNC-RP already provide us this information beforehand. As such, a future approach could determine the support layout with respect to the worst-known conditions, rather than the worst-possible conditions as is currently done.
Reference


