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Solutions of Systems of First-Degree Equations and Inequalities in Two Variables

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SOLUTIONS OF SYSTEMS OF FIRST-DEGREE

EQUATIONS AND INEQUALITIES IN TWO VARIABLES

(TITLE)

BY

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PREFACE

It is the intent of the writer that the contents of this paper be used along with other units forming a first course in algebra. The writer assumes that a proper groundwork has been laid with regard to the normal sequence of topics covered in a first course in algebra. He especially assumes that adequate graphing of solution sets of first-degree equations in one and two variables and first-degree inequalities in one variable has been accomplished.

Owing to the familiarity of the writer with the algebra text written by Dolciani, Berman, and Freilich (see bibliography), he will, for the most part, be using the terminology employed in this text with the belief that this terminology does not differ significantly from that employed in other modern texts currently in use. However, definitions of terms which students already should have been exposed to in previous work, along with those definitions that students will need to proceed with the material contained in this paper, will be given where appropriate.

Although the contents of the paper are expected to be used in conjunction with other units making up a first course in algebra, this is not to say that the material is to be used at any particular grade level. From what the writer has been able to ascertain from the somewhat meager amount of material on the subject which was at his disposal, the teaching of solutions of systems of first-degree equations and inequalities in two variables along with related materials has been successfully completed with accelerated eighth graders and even highly gifted seventh graders. However, the great majority of modern textbooks include this particular material in an algebra text which is meant for ninth graders. As for the future, if The Cambridge Conference on School Mathematics continues to influence teachers, educators, and mathematicians as greatly as it has thus far, solutions of systems of first-degree equations and inequalities in two variables will be commonly taught to pupils in grade six, and possibly in even lower grades.

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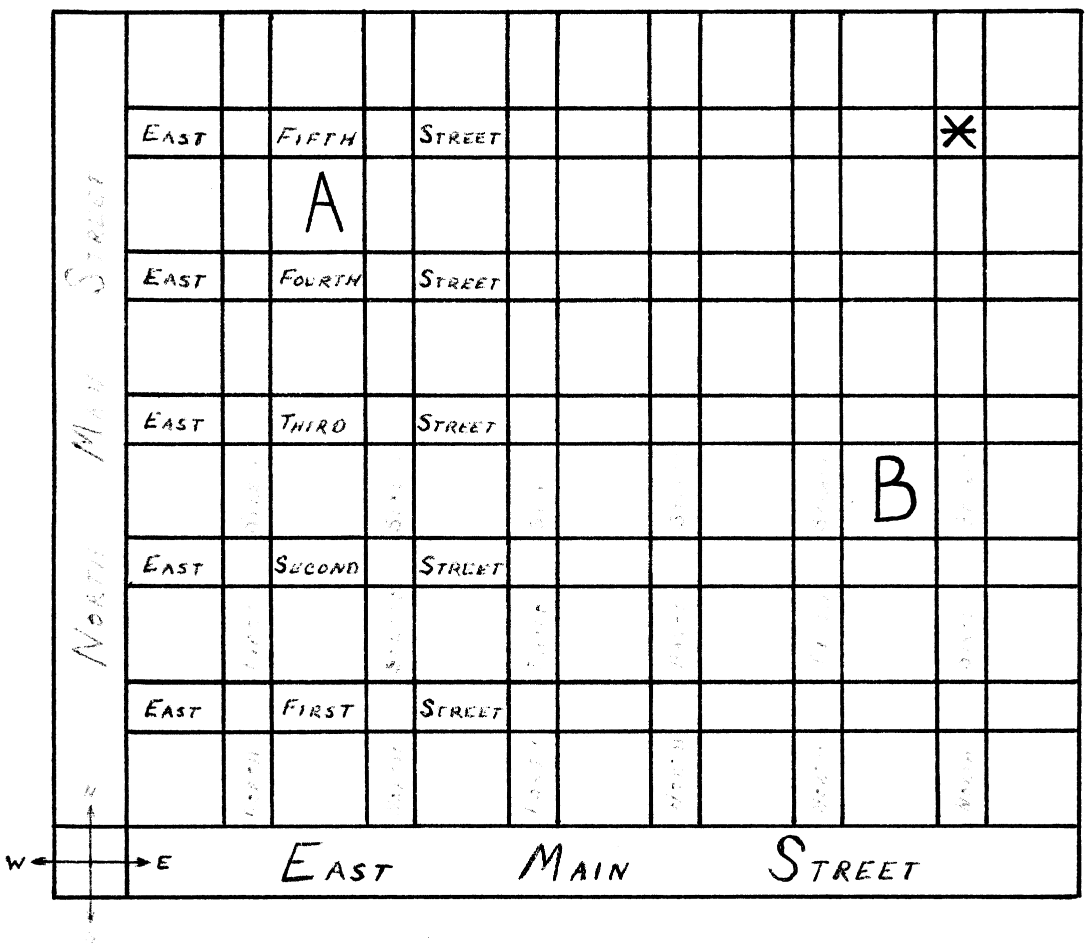
CHAPTER I

GRAPHICAL SOLUTIONS OF FIRST-DEGREE EQUATIONS



The diagram above represents a section of Blocksville, U.S.A. As can be seen from the diagram, the streets run north-south and east-west with Robert and Roberta Black living on block A and Steve and Stella White living on block B. Monday thru Friday, Robert leaves his home on block A and drives east on East

Fifth Street to his job in the suburbs. Also, during the same period of time, Steve leaves his home on block B and drives north on North Sixth Street to his job in another suburb. At the intersection of these two streets, Robert and Steve, who are good friends, wave at each other every time they happen to see one another pass through. The student should observe that the intersection of these two streets, which is a part of both streets, is at the spot marked with the *. If the student can now imagine the streets as being representations of lines and this section of town as being a representation of a portion of a coordinate plane, he should have a good idea as to what a mathematician means when he is speaking of two straight lines in the same plane intersecting in a point.[1]

Streets running north and south represent the graphs of equations with X = C (a whole no. constant), streets running east and west represent the graphs of equations with Y = C (a whole no. constant), and East Main Street and North Main Street represent the X-axis and Y-axis, respectively.

What would be the coordinates of the point of intersection of the two lines represented by East Fifth Street and North Sixth Street (the intersection where Robert and Steve wave to each other)? If the student

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[1]Eugene D. Nichols, Modern Elementary Algebra (New York, 1965), p. 269.

remembers to give the X-coordinate first and the Y-coordinate second he should reply (6,5). If Roberta drives south on North First Street and Stella drives west on East Third Street, as each goes shopping downtown, what would be the coordinates of the point of intersection of the two lines represented by these two streets?

On Saturdays, Robert and Steve go in separate cars to a golf course far out in a suburb east of town. As the golf course is quite large, both East Second Street (which Steve uses to drive to the course) and East Fourth Street (which Robert uses to drive to the course) pass it. Again the student is asked to think of streets as being representations of lines. What would be the coordinates of the point of intersection of these two lines? Give up? Well, if the student recognizes that streets which run in the same direction normally do not cross, then there could be no point of intersection for the lines represented by East Second Street and East Fourth Street. When a mathematician says that two _parallel lines_ are two straight lines in the same plane which never intersect, this should now have meaning for the student.[2]

On Wednesdays, Stella leaves home about an hour after Steve and drives north on North Sixth Street, as Steve had done earlier, to a beauty shop which is across

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[2] _Ibid._, p. 273.

the street from where Steve works in the suburbs. Once more the student is asked to think of streets as being representations of lines. What would be the coordinates of the point of intersection of this line with itself? If the student answers that there would be numerous points of intersection since the routes taken by Steve and Stella are identical, he would indeed be observant! When a mathematician says that two straight lines coincide, the student should understand that this is the same thing as saying that the graphs of two linear equations in the same variables result in the same straight line.[3] (The student should remember that an equation of degree one is called a linear equation.[4]) Therefore, it should be seen that two coincident lines would have an unlimited or infinite number of points in common.[5]

"Because two [linear] equations [in two variables] impose two conditions on the variables at the same time, they are called a system of simultaneous equations."[6] To solve this system of equations, it is necessary to find the ordered pairs of numbers which satisfy both equations. One method for doing this is to graph the solution set of each equation (the set of all ordered

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[3] Ibid., p. 272.

[4] Mary P. Dolciani, Simon L. Berman, and Julius Freilich, Modern Algebra, Structure and Method: Book I (Boston, 1965), p. 264.

[5] Ibid., p. 368.

[6] Ibid.

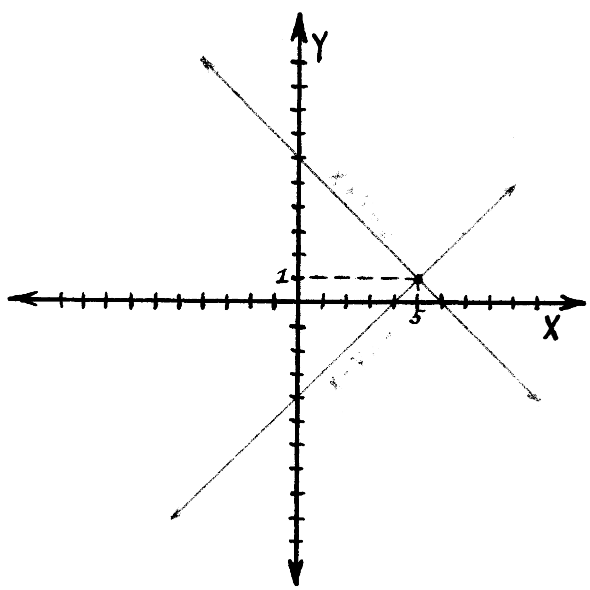
pairs satisfying the equation)[7] and then determine by inspection the coordinates of the point of intersection of the two lines, <u>if they intersect</u>! The student should keep in mind that the two lines might be parallel!

If the pair of linear equations

$$X + Y = 6$$

$$X - Y = 4$$

were graphed on the same coordinate plane, the result would appear as indicated below:



What would be the coordinates of the point of intersection? If the student's answer to the previous question is (5,1), this can be verified by substituting these numerals for the proper variables in each of the linear equations and noting that true statements would

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[7]*Ibid.*, p. 334.

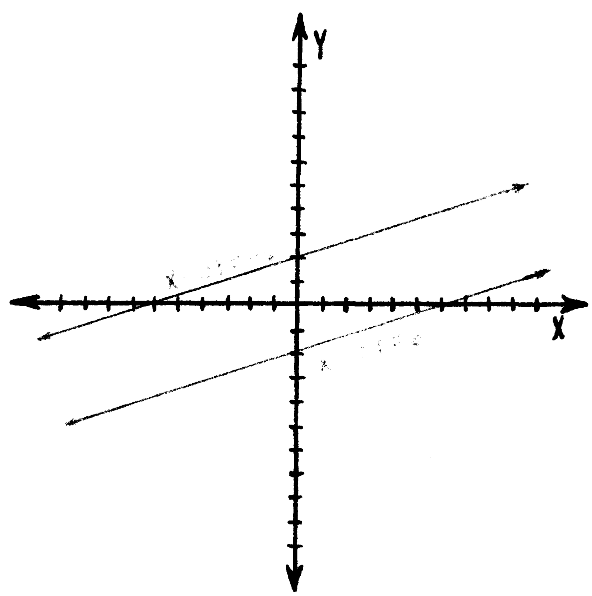
result. If two lines intersect, their equations have one common solution and are referred to as consistent and independent equations.[8]

If the pair of linear equations

$$X - 3Y = 6$$

$$X - 3Y = -6$$

were graphed on the same coordinate plane, the result would appear as indicated below:



As the lines seem to be parallel (the student might more readily accept this if he were to trace the lines on another sheet of paper and then rotate the paper over the above graph such that the opposite lines would be matched up), they would not intersect and thus there would be no common solution for the two equations. If

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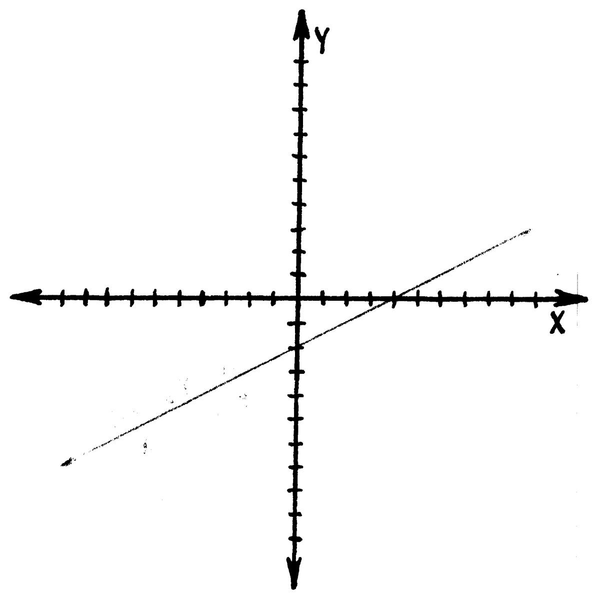
[8] *Ibid.*, p. 368.

two lines are parallel their equations have no common solution and are referred to as inconsistent equations.[9]

If the pair of linear equations

$$X - 2Y = 4$$
$$3X - 6Y = 12$$

were graphed on the same coordinate plane, the result would appear as indicated below:



As can be seen, the graphs of the two equations are the same straight line. This would mean that there are an infinite number of common solutions to the two equations. The student should verify several of these common solutions by picking ordered pairs of numbers which satisfy one equation and then, by substitution, see that they would satisfy the other equation. If the

[9] Ibid.

student were to divide each member of the second equation by 3 (or multiply each member of the first equation by 3), he would find that he has identical equations--which would mean that any ordered pair satisfying one equation must satisfy the other. In other words, if one equation can be made identical to the other equation by performing a multiplication or division on either or both equations of the system, their graphs _must_ coincide. If two lines do coincide, their equations have an infinite number of common solutions and are referred to as _consistent_ and _dependent_ equations.[10] Since the equations $X - 2Y = 4$ and $3X - 6Y = 12$ have the same solution set, they are referred to as _equivalent equations_.[11]

The student should graph the following systems of equations, determine the coordinates of the point of intersection (if they intersect), and tell whether they are _consistent_ or _inconsistent_ and _dependent_ or _independent_:

1.) $X + Y = 7$ $X - Y = 3$	2.) $2X - Y = 14$ $X + Y = 10$	3.) $2X + 2Y = 8$ $3X + 3Y = 12$
4.) $X - Y = -5$ $X - Y = 6$	5.) $X - 2Y = 4$ $3X - 6Y = 12$	6.) $Y - 2X = 4$ $Y + X = 1$

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[10] _Ibid._

[11] _Ibid._, p. 81.

7.) $Y + X = 0$	8.) $Y - \|X\| = 0$	9.) $X + 2Y = -4$
$Y + X = 2$	$2Y - X = 6$	$7X - 2Y = -4$

CHAPTER II

ALGEBRAIC SOLUTIONS OF FIRST-DEGREE EQUATIONS

Except for exercise 9 in the last chapter, all other examples and exercises involving systems of consistent and independent equations have resulted in a solution where both coordinates are whole numbers. As the student has been working in the set of real numbers, numbers which are either rational or irrational,[12] he should not continuously expect the coordinates of the point of intersection of the two lines to be whole numbers. If the student were to attempt a graphical solution for every system of equations that he is required to solve in the future, he would soon come to the realization that the coordinates of the point of intersection are not always easily discernible.

This then, should bring the question to the student's mind as to whether or not there might be alternate methods for the solution of systems of equations which would _always_ result in accurate answers. There are such methods. One, called the _addition-subtraction_

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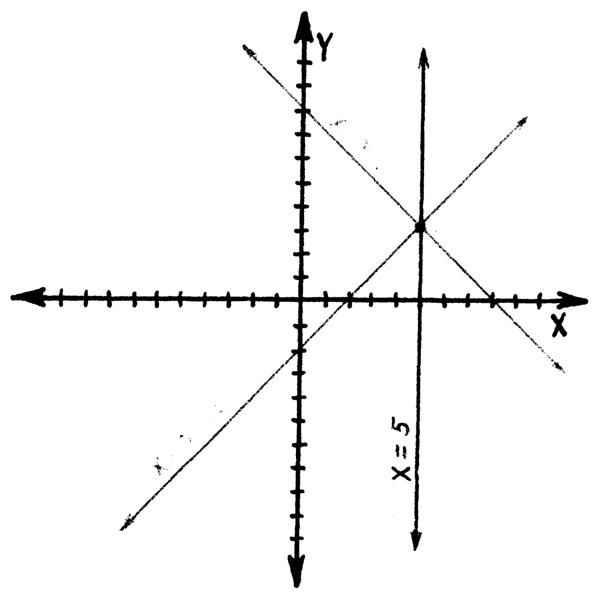
[12] J. Houston Banks, Max A. Sobel, and William Walsh, _Algebra: Its Elements and Structure, Book I_ (St. Louis, 1965), p. 64.

method,[13] is demonstrated in the following:

$$X + Y = 8$$
$$\underline{X - Y = 2}$$
$$2X = 10$$ (by using the addition property of equality)
$$X = 5$$ (by using the division property of equality)

At this point the student is asked to turn his attention to the graph of the preceding system of equations X + Y = 8 and X - Y = 2, and, to especially note the graph of X = 5 and its relationship to the graph of the system.

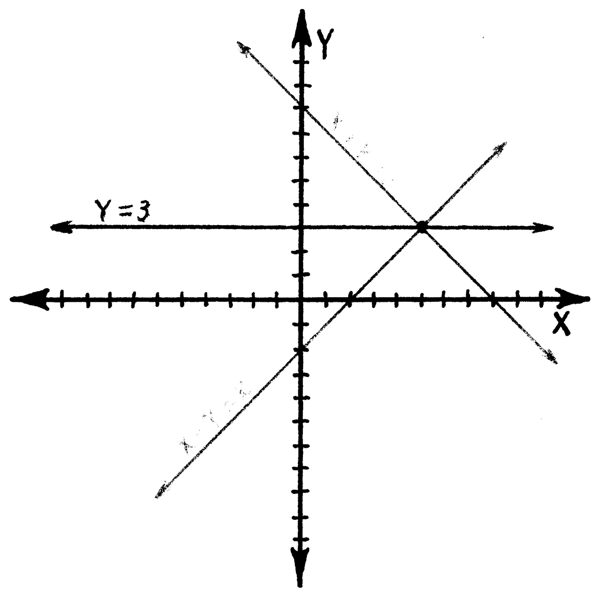


Now, if 5 is substituted for X in either of the original equations of the system:

[13]Howard F. Fehr, Walter H. Carnahan, and Max Beberman, Algebra, First Course (Boston, 1965), p. 256.

$X + Y = 8$	<u>or</u>	$X - Y = 2$
$(5) + Y = 8$		$(5) - Y = 2$
<u>$Y = 3$</u>		<u>$3 = Y$</u>

Again the student is asked to turn his attention to the graph of the system $X + Y = 8$ and $X - Y = 2$, but, this time he should especially note the graph of <u>$Y = 3$</u> and its relationship to the graph of the system.



As can be seen from the preceding two graphs, using the addition-subtraction method in solving the system of equations created another system of equations consisting of the graphs of the equations $X = 5$ (the <u>vertical</u> line) and $Y = 3$ (the <u>horizontal</u> line) which pass through the point of intersection of the two lines of the first system.[14] By observing this, it can readily be seen that

[14]Dolciani, p. 370.

the solution set is $\{(5,3)\}$ for both systems. As the two systems of equations [(1.) X + Y = 8 and X - Y = 2, (2.) X = 5 and Y = 3] have the same solution set, they are referred to as equivalent systems.[15]

If the student would again turn his attention to the original system of equations, the subtraction property of equality will first be utilized instead of the addition property. This is being done to show the student that either property could have been used first. Thus (if there is a choice), it is up to the student as to which he prefers.

$$\begin{aligned} X + Y &= 8 \\ \underline{X - Y} &\underline{= 2} \\ 2Y &= 6 \end{aligned}$$ (by using the subtraction property of equality)

$\underline{\underline{Y = 3}}$ (by using the division property of equality)

As was illustrated before, the graph of Y = 3 is the horizontal line passing through the point of intersection of the lines of the system X + Y = 8 and X - Y = 2.

Now, if 3 is substituted for Y in either of the original equations of the system:

$$\begin{aligned} X + Y &= 8 \\ X + (3) &= 8 \\ \underline{\underline{X \quad}} &\underline{\underline{= 5}} \end{aligned} \qquad \underline{\text{or}} \qquad \begin{aligned} X - Y &= 2 \\ X - (3) &= 2 \\ \underline{\underline{X \quad}} &\underline{\underline{= 5}} \end{aligned}$$

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[15] Henry Van Engen and Others, Algebra (Glenview, Illinois, 1966), p. 524.

Again, the graph of <u>X = 5</u> is the vertical line passing through the point of intersection of the lines of the system X + Y = 8 and X - Y = 2.

Considering, once more, the equivalent system Y = 3 and X = 5, it is quite easy to list the solution set as $\{(5,3)\}$ for this system; but, the student should remember that the system X + Y = 8 and X - Y = 2 would also have the <u>same</u> solution set.

If an attempt is made to solve the system of equations given below by using the addition-subtraction method:

$$\begin{array}{l} 6X - Y = 11 \\ \underline{3X + 2Y = 8} \\ 9X + Y = 19 \end{array}$$ (by using the addition property of equality)

<u>or</u>

$$\begin{array}{l} 6X - Y = 11 \\ \underline{3X + 2Y = 8} \\ 3X - 3Y = 3 \end{array}$$ (by using the subtraction prop. of equality)

The resulting equation in both instances is not one whose graph is either a horizontal or vertical line passing through the point of intersection of the lines determined by the two equations of the system. Consequently, the coordinates of the point of intersection are no closer to being identified than before the computations.

In order to eliminate a variable by using the addition-subtraction method in this situation, it is first necessary to use the multiplication property of equality to make the coefficient of a variable in one equation of the system equal to (or equal to the opposite of) the corresponding coefficient in the other equation of the system.

In the previous example, using the multiplication property of equality on the first equation:

$$\begin{array}{lll} 6X - Y = 11 \rightarrow & 12X - 2Y = 22 & \text{(mult. prop. of equality--by 2)} \\ 3X + 2Y = 8 & \underline{3X + 2Y = 8} & \\ & 15X = 30 & \text{(add. prop. of equality)} \\ & \underline{\underline{X = 2}} & \text{(div. prop. of equality)} \end{array}$$

If 2 is substituted for X in either of the original equations:

$$\begin{array}{rcr} 6X - Y = 11 & \underline{\text{or}} & 3X + 2Y = 8 \\ 6(2) - Y = 11 & & 3(2) + 2Y = 8 \\ 12 - Y = 11 & & 6 + 2Y = 8 \\ \underline{\underline{1 = Y}} & & 2Y = 2 \\ & & \underline{\underline{Y = 1}} \end{array}$$

This would mean that the solution set is $\{(2,1)\}$.

Returning, once more, to the system of equations $6X - Y = 11$ and $3X + 2Y = 8$, the multiplication property of equality will be used on the second equation of the system. This is being done to show the student that the multiplication property can be used with either equation of the system to effect satisfactory results. Again, the choice is left to the student.

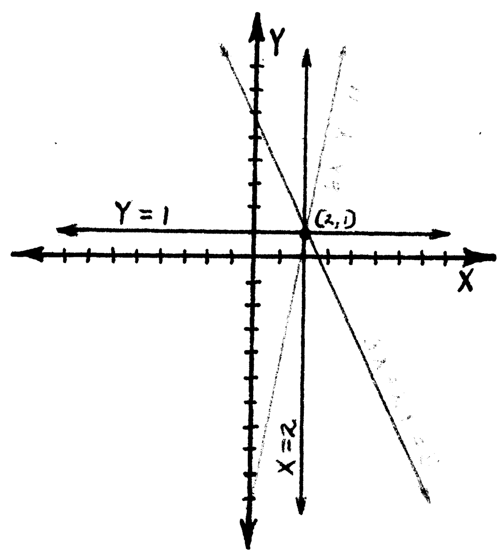
$$
\begin{array}{lll}
6X - Y = 11 & & 6X - Y = 11 \\
3X + 2Y = 8 & \longrightarrow & \underline{6X + 4Y = 16} \text{ (mult. prop. of equality--by 2)} \\
& & -5Y = -5 \text{ (sub. prop. of equality)} \\
& & \underline{\underline{Y = 1}} \text{ (div. prop. of equality)}
\end{array}
$$

If 1 is substituted for Y in either of the original equations:

$$
\begin{array}{lcl}
6X - Y = 11 & \underline{\text{or}} & 3X + 2Y = 8 \\
6X - (1) = 11 & & 3X + 2(1) = 8 \\
6X = 12 & & 3X + 2 = 6 \\
\underline{\underline{X = 2}} & & 3X = 6 \\
& & \underline{\underline{X = 2}}
\end{array}
$$

We again have the solution $\{(2,1)\}$.

To remind the student what is happening graphically when the addition-subtraction method is employed, the graph of the previous system of equations as well as the graphs of the horizontal and vertical lines passing through the point of intersection of the system follow:



Thus far a check has not been used in the addition-subtraction method of solving systems of equations. The reason is that the coordinate of the point of intersection which was first determined was substituted for the proper variable in both of the original equations. When this gave identical results for the other coordinate, it meant that the proper coordinates for the point of intersection had been discovered. Actually, this could be used as a check; but, another method is generally suggested. In the previous example, if 1 were substituted for Y in the first equation only, the result should be 2 for the value of X. The check then would be:

$$\begin{aligned} 6X - Y &= 11 \\ 6(2) - (1) &= 11 \\ 12 - 1 &= 11 \\ 11 &= 11 \checkmark \end{aligned} \qquad \text{and} \qquad \begin{aligned} 3X + 2Y &= 8 \\ 3(2) + 2(1) &= 8 \\ 6 + 2 &= 8 \\ 8 &= 8 \checkmark \end{aligned}$$

The student should remember to substitute the values he has obtained for X and Y in both original equations of the system (as shown); otherwise, he may not have the coordinates of the point of intersection. For example, consider the following:

$$\begin{aligned} 2X - Y &= 13 \qquad & 2X - Y &= 13 \\ X + 2Y &= 3 \longrightarrow & \underline{2X + 4Y} &\underline{= 3} \end{aligned}$$ (mult. prop. of equality--by 2)

$$-5Y = 10$$

$$\underline{\underline{Y = -2}}$$

If -2 is substituted for Y in the equation $2X + 4Y = 3$, the result is:

$$\begin{aligned} 2X + 4Y &= 3 \\ 2X + 4(-2) &= 3 \\ 2X - 8 &= 3 \\ 2X &= 11 \\ \underline{\underline{X}} &\underline{\underline{= 11/2 \text{ or } 5\tfrac{1}{2}}} \end{aligned}$$

If a check were attempted by using the equations $2X - Y = 13$ and $2X + 4Y = 3$:

$$\begin{aligned} 2X + 4Y &= 3 \\ 2(11/2) + 4(-2) &= 3 \\ 11 - 8 &= 3 \\ 3 &= 3 \checkmark \end{aligned} \qquad \begin{aligned} 2X - Y &= 13 \\ 2(11/2) - (-2) &= 13 \\ 11 + 2 &= 13 \\ 13 &= 13 \checkmark \end{aligned}$$

Thus, the student might say that the solution set is $\left\{(5\tfrac{1}{2}, -2)\right\}$!

But, if the check were attempted on the other <u>original</u> equation $X + 2Y = 3$:

$$\begin{aligned} X + 2Y &= 3 \\ (11/2) + 2(-2) &= 3 \\ 5\tfrac{1}{2} - 4 &= 3 \\ 1\tfrac{1}{2} &= 3 \quad (\underline{\text{false}}!) \end{aligned}$$

What has gone wrong? The student's attention should now be drawn to where the multiplication property of equality was used with the equation $X + 2Y = 3$ to obtain $2X + 4Y = 3$. The right member of the equation has not been multiplied by 2 as has the left member. Therefore, although there were no errors made in succeeding calculations, the solution set for the original system of equations has not been found.

Another method commonly used for solving systems of linear equations is the <u>substitution method</u>.[16] As might be inferred from the preceding statement, the substitution method involves the use of the <u>substitution principle</u>.[17] This method is presented in addition to the addition-subtraction method as a means of solving systems of linear equations because it sometimes is simply easier to use than the other method.

In order to use the substitution method, it is first necessary to solve one or the other of the equations in the system for one variable in terms of the other (unless one of the equations is given in that form). For the following system then:

$$2X + Y = 8$$
$$4X - 2Y = 4$$

$Y = 8 - 2X$ (solving first equation for Y in terms of X)

It is next necessary to substitute this expression for Y in the other equation:

$$4X - 2(8 - 2X) = 4$$

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[16]Fehr, p. 254.

[17]Dolciani, p. 71.

Now, it becomes necessary to solve the resulting equation for X:

$$4X - 16 + 4X = 4$$
$$8X = 20$$
$$\underline{\underline{X = 5/2 \text{ or } 2\tfrac{1}{2}}}$$

Finally, the value obtained for X must be substituted in either of the original equations:

$$2X + Y = 8$$
$$2(5/2) + Y = 8$$
$$5 + Y = 8$$
$$\underline{\underline{Y = 3}}$$

Checking:

$$2X + Y = 8 \qquad 4X - 2Y = 4$$
$$2(5/2) + (3) = 8 \qquad 4(5/2) - 2(3) = 4$$
$$5 + 3 = 8 \qquad 10 - 6 = 4$$
$$8 = 8 \checkmark \qquad 4 = 4 \checkmark$$

This indicates the solution is $\{(5/2,3)\}$.

In case the student is wondering why the first equation of this system was solved for Y in terms of X instead of some other solution involving one variable in terms of the other, he should closely examine both equations and observe that this procedure was just a little easier than the others which could have been tried. For example, if the second equation had been solved for X:

$$4X - 2Y = 4$$

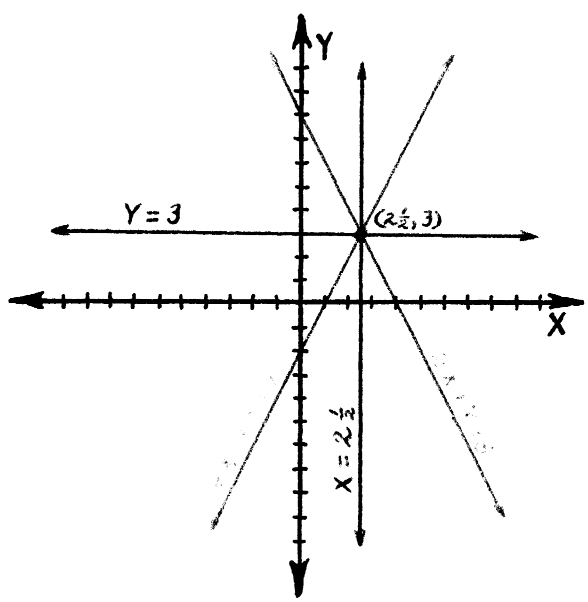
$$4X = 4 + 2Y$$

$$X = 2(2 + Y)/4$$

$$X = (2 + Y)/2 \text{ or } 1 + Y/2$$

The rest of the calculations are left for the student to perform. He should then consider which of the two (or more) procedures--solving the first equation for Y in terms of X and then proceeding or solving the second equation for X in terms of Y and then proceeding--he would prefer.

Once more a graph is given of the foregoing work that the student might better keep in mind the various relationships existing between the original system and the equivalent system of equations.



The student may work the following exercises by using either the addition-subtraction method or the substitution method of solving systems of linear equations (whichever appears to be easier).

1.) $2X + Y = 4$
$X - Y = -1$

2.) $3X - 2Y = 5$
$X + 3Y = 9$

3.) $X = 3$
$2X + 3Y = 9$

4.) $3X + Y = 7$
$X + 3Y = 9$

5.) $5X + 2Y = 1$
$4X + 3Y = -2$

6.) $3X - 2Y = 1$
$5X + 3Y = -11$

7.) $2X + 3Y = 3$
$X + Y = 0$

8.) $X - 3Y = 1$
$5X - 7Y = -3$

9.) $4X + Y = 9$
$3X + 2Y = 13$

10.) $2X - 9Y = 3$
$X - 3Y = 6$

11.) $2X + Y = -5$
$6X + 3Y = -15$

12.) $2X - 3Y = 11$
$4X + Y = 8$

13.) $2X - Y = -3$
$4X - Y = -2$

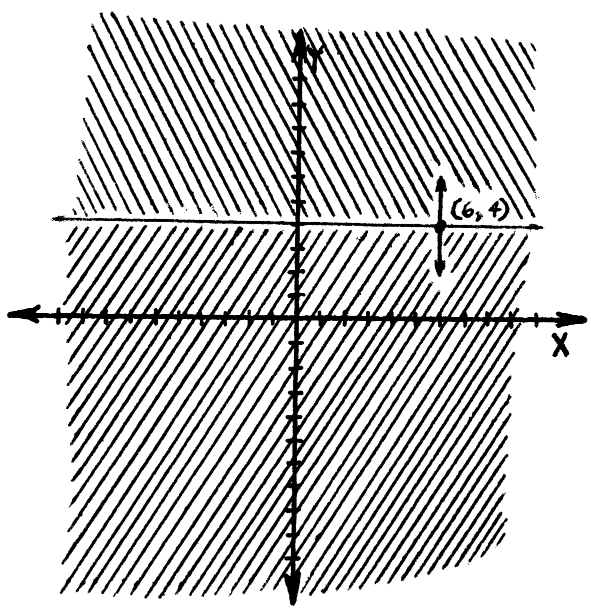
14.) $3X - 2Y = -7$
$2X - 5Y = 10$

15.) $9X - 3Y = 5$
$3X - Y = 4$

CHAPTER III

GRAPHICAL SOLUTIONS OF FIRST-DEGREE INEQUALITIES

Although the student should have graphed an inequality in two variables in previous work, the concepts behind this are given below that it might refresh his memory and help him in understanding the ideas presented later in the chapter.



In the above figure the horizontal line (graph of $Y = 4$) divides the coordinate plane into two regions (the two shaded portions). Each region is referred to as a half-plane.[18] The line, which is the graph of

[18] Ibid., p. 351.

Y = 4, is said to be the boundary of the two half-planes.[19] The idea is that any line in a plane would divide the plane into two half-planes with the line being considered the boundary of both half-planes. However, the line can be considered a part of either or both half-planes. If, in graphing, it is not desired to include the boundary line as a part of some half-plane, this line is dashed (see p. 26).

If the student would start at any point on the line, say (6,4), and move vertically upward, he should note that the Y-coordinate increases. If the student were to move vertically downward from this same point, he would observe that the Y-coordinate decreases. In both cases, moving either upward or downward, the X-coordinate remains the same. The half-plane above the line consists of points for which $Y > 4$, and is the graph of that inequality. The half-plane below the line is the graph of $Y < 4$. The half-plane above the line along with the boundary line forms the graph of $Y \geq 4$, whereas the boundary line and the half-plane below it is the graph of $Y \leq 4$.

In the following figure, the vertical line (graph of $X = 5$) has divided the coordinate plane into two half-planes. The half-plane to the right of the line is the graph of $X > 5$ and the half-plane to the left of the line is the graph of $X < 5$. Thus, the boundary line and

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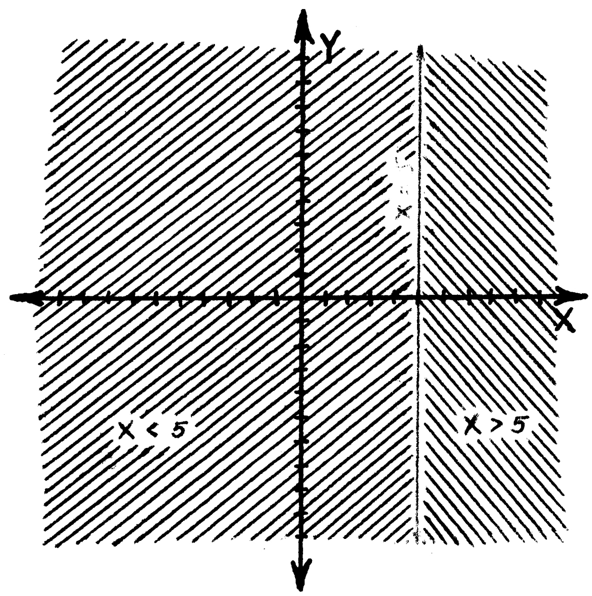
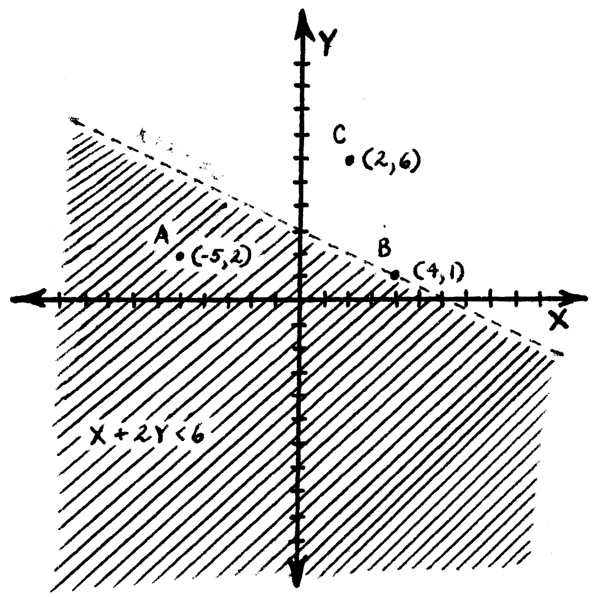
[19] Ibid.

the half-plane to the right of it form the graph of $X \geq 5$; and, the boundary line and the half-plane to the left of it would form the graph of $X \leq 5$.



The inequality $X + 2Y < 6$ has been graphed on the following coordinate plane. The student should verify this by picking several points in the shaded portion and substituting their coordinates in the inequality $X + 2Y < 6$. For example, using pt. A, we would have $(-5) + 2(2) < 6$. Which is true! He should then pick points on or above the dashed line and verify that their coordinates, if substituted in the inequality, create false statements. For example, using pt. B, we would have $(4) + 2(1) < 6$. Which is false! Using pt. C, we would have $(2) + 2(6) < 6$. Which is false! The dashed line, as the student should remember, is used to indicate that the line is not a part of the graph of $X + 2Y < 6$. If $X + 2Y < 6$ were graphed, the line would be solid to indicate that it would be a part of the graph. <u>What</u> would be the graph of $X + 2Y \geq 6$?

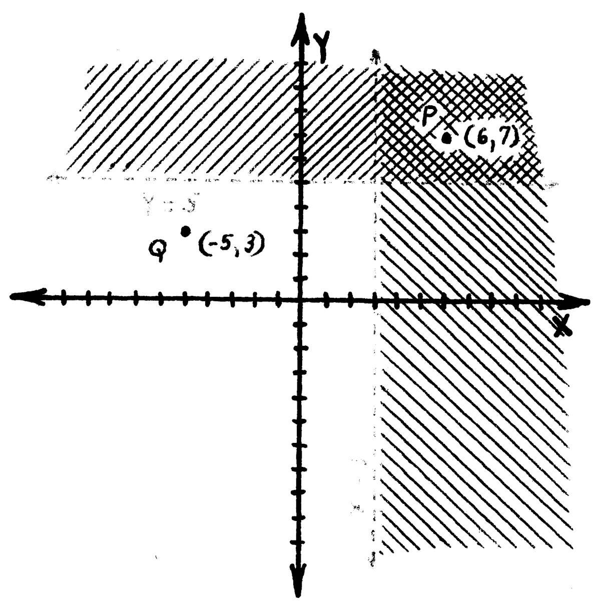


The system of inequalities

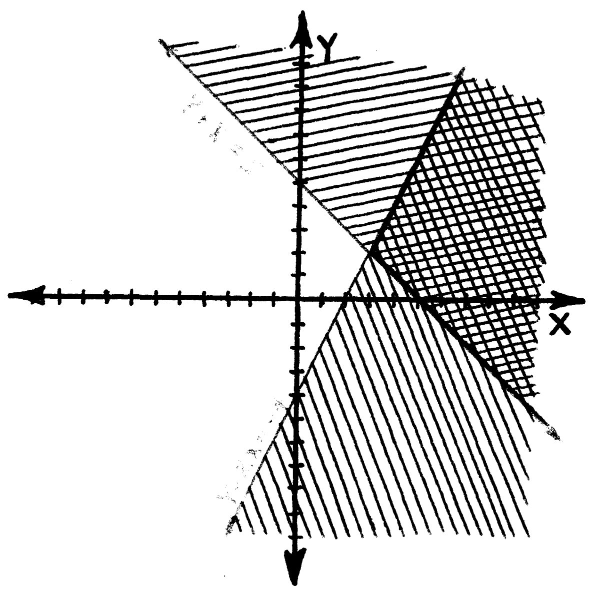
$$Y > 5$$

$$X > 3$$

graphed on the same coordinate plane would appear:



The points in the upper right-hand section (double-shaded) of the plane have coordinates which satisfy both inequalities. That is, these are the points for which $Y > 5$ and $X > 3$. For example, using pt. P, by substitution in the inequalities we have $(7) > 5$ and $(6) > 3$; which, obviously, are true. Nowhere else in the plane will the coordinates of the points satisfy both inequalities. For example, using pt. Q, we have $(3) > 5$ and $(-5) > 3$, neither of which is true. The student should again pick several other points, including points on either of the two dashed lines, to satisfy himself that the coordinates of the points in the double-shaded portion are the only ones which will satisfy the two inequalities.



The graph of the system of inequalities $Y - 2X \leq -4$ and $Y + X \geq 5$ appears above. It was obtained by first graphing the equations $Y - 2X = -4$ and $Y + X = 5$. Then the half-planes above $Y + X = 5$ and below $Y - 2X = -4$ were shaded. The double-shaded portion of the coordinate plane along with the darkened portions of the two lines then becomes the graph of the solution set of the two inequalities. It is left to the student to <u>verify</u> that the double-shaded portion and the darkened portions of the two lines is the solution set.

The student should now graph the following systems of inequalities to determine their solution sets:

1.) $X > 7$ $Y < -4$	4.) $Y - 2X < -8$ $Y \geq -3$	7.) $X > -7$, $Y < 6$, $X < 5$, $Y > -4$
2.) $Y \geq -5$ $X \geq 0$	5.) $Y - 3X \leq 6$ $Y - 3X \geq -6$	8.) $Y + X/2 \geq -4$, $Y \leq 5$, $X \leq 3$
3.) $Y \geq X$ $Y + X \leq -2$	6.) $Y - 3X \geq 6$ $Y - 3X \leq -6$	9.) $Y \geq \|X\| - 2$ $Y \leq -\|X\| + 3$

BIBLIOGRAPHY

Books

Banks, J. Houston, Sobel, Max A., and Walsh, William. Algebra: Its Elements and Structure, Book I. St. Louis: McGraw-Hill, Inc., 1965.

Dolciani, Mary P., Berman, Simon L., and Freilich, Juluis. Algebra, Structure and Method: Book I. Boston: Houghton Mifflin Company, 1965.

Engen, Henry Van, and Others. Algebra. Glenview, Illinois: Scott, Foresman and Company, 1966.

Fehr, Howard F., Carnahan, Walter H., and Beberman, Max. Algebra, First Course. Boston: D. C. Heath and Company, 1965.

Nichols, Eugene D. Modern Elementary Algebra. New York: Holt, Rinehart and Winston Inc., 1965.

Reports

College Entrance Examination Board. Program for College Preparatory Mathematics. Report of the Commission on Mathematics. New York: College Entrance Examination Board, 1959.

Educational Services Incorporated. Goals for School Mathematics. The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company, 1963.