Teaching Problem-Solving in the High School

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TEACHING PROBLEM-SOLVING

IN THE HIGH SCHOOL

(TITLE)

BY

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YEAR

I HEREBY RECOMMEND THIS PLAN B PAPER BE ACCEPTED AS
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INTRODUCTION

Just as skill in reading is necessary for rapid learning in nearly all fields, so the teaching of problem-solving in the high school is basic to all problem-solving in later life. The adult with no background in how to solve "coin-problems" will also be stumped with what to do with a leaky faucet.

However, it has been the experience of the writer, both personally, and in conversation with other mathematics teachers, that the development of an ability to do "problem-solving" is probably the most difficult of all teaching assignments. This is true not only at the high school level, but also in the junior high school, and on down into grade school arithmetic.

The problem is not new. Descartes attempted to devise a universal method for solving all problems. Leibnitz, also, formulated the idea of a perfect method. But the search for a universal perfect method of problem-solving was no more successful than the quest for the philosopher's stone, which would turn base metals into gold.²

Nearly every textbook which contains a unit on problem-solving includes a "Methodical Procedure for Solving Problems" which begins:

"1. Read the problem carefully."

The pattern may vary slightly from text to text, but essentially the steps are:

1. Understand the problem.
2. Choose the known and required quantities from the given data.
3. Set up a relation between these quantities.
4. Solve the relationship.
5. Check the answer.

But even with such a pattern available, many students seemingly cannot solve problems. This would lead one to look elsewhere for the answer to the question, "Why can't high school students solve word problems?"

The purpose of this paper is to point out some of the other factors involved and some suggested ways of overcoming them.
CHAPTER I

DEFINITION OF PROBLEM-SOLVING

What is meant by the term "problem-solving"?

Usually, problem-solving is taken to mean the solution of "word problems" or "statement problems" such as coin problems, age problems, work problems and the like. This is the context in which the term will be used throughout this paper.

Stripped to the very essentials, the elements of a problem are: 2

1. There is something you want, and
2. You don't know how to get it.

By definition, each problem must be unique; that is, it must contain some new obstacle. If it can be "solved" by any automatic or previously-learned response, it ceases to be a problem, and is simply repetitive drill. 3

It has been pointed out 4 that mathematics does not have a monopoly on problem-solving. It is a universal

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phenomenon. The cancer researcher, for example, is seeking to solve a problem—a big problem. But problems in any and all fields have one thing in common: solving them is a progression from the given data to the desired goal by studying both the data and the goal, conjecturing a possible connection between them, and selecting the proper means for establishing that connection.

Since problems in all fields of endeavor are solved by similar means, it is imperative that students be able to solve problems—not only to pass a course in mathematics, but so that the techniques learned in the classroom may be applied to everyday problems. It also imposes upon the teacher the necessity of teaching in such a way that this transfer may be accomplished.
CHAPTER II

REASONS FOR THE DIFFICULTY

What reasons can be advanced for the difficulty experienced by "normal" high school boys and girls in the area of problem-solving? There seem to be several reasons.

One reason children cannot solve problems is because they cannot read—or at least cannot understand what they do read. A correlation study between algebra aptitude scores and reading scores (both on standardized tests) for 102 entering high school freshmen, gave a correlation figure of 0.74, which is considered substantially significant. (The writer recognizes that it can be argued that both of these abilities are tied closely to intelligence. However, the fact remains; those who cannot read cannot work problems.)

David H. Russell\(^6\) states:

Much of the humor classed as "schoolboy boners" comes under the head of wrong or inadequate concepts. The boy who thought an average was "something hens

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lay eggs on" (He had read the arithmetic problem: Farmer Jones' hens laid 24 eggs on Sunday, 32 eggs on Monday, 28 eggs on Tuesday, etc. How many eggs did they lay on the average during the week?) was suffering from an inaccurate concept.

Children suffer inaccurate concepts not only in English, but also in the language of mathematics. Much of the aversion to mathematics may be caused by not fully understanding the basic concepts of mathematics. We never like something we cannot understand. 7

Many of these fuzzy concepts are undoubtedly the result of poor teaching. E. A. Bond 8 points out that in the two decades immediately after the turn of the century there was little or no improvement in children's abilities to do computational arithmetic, but during the same period their problem-solving ability greatly decreased. This change he attributes to the advent of standardized tests. The first tests marketed were developed for testing computational skills (since these were the easiest to devise). As a consequence, teachers emphasized computing almost exclusively, and problem-solving fell by the wayside.

Guy M. Wilson 9 makes the statement that in a typical sixth or seventh grade text the examples are nothing but

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disguised drill and "no one really cares what the answers are."

Poor teaching and poor textbooks lead naturally to a general lack of interest in mathematics. This deplorable state of affairs is aggravated by many adults who "never liked math myself." Even the "comics" may add fuel to the fire by depreciating the role of mathematics. 10 Some of this rubs off on boys and girls.

Some students, though, do have genuine emotional problems where mathematics is concerned. Mary Tulock 11 describes some pupils who were so tense after traumatic (mathematics) experiences that they seemed simply paralyzed. So rigid was the set against mathematics that some were convinced they could never learn mathematics.

These, then, are a few of the problems to be faced in the teaching of problem-solving. Since in life outside of school, people figure only when they want an answer, and they figure only when there is a background of experience, the school must furnish this experience. This is the task of the teacher of problem-solving.


11 Mary K. Tulock, "Emotional Blocks in Mathematics," The Mathematics Teacher, L (December, 1957), 272-76.
CHAPTER III

TREATMENT OF THE SYMPTOMS

To be able to handle problems effectively, the student must be retaught how to read. Every since the student first learned how to read, teacher after teacher has been trying to teach him to increase his speed of reading. We must do the reverse. He must be taught to read slowly, to pause at commas or at the end of a single idea, to consider what he has read and decide what it means, to go back and re-examine it as many times as necessary. Bright students in particular need to be slowed up.12

Very slow readers, on the other hand, might be helped by classes in remedial reading (administered by the English department). Where ability grouping is feasible, special selection of reading materials is advised. There should also be practice in reading skills related to problem-solving.13

Any subject has a specialized vocabulary. The meanings of such words must be carefully defined as the language is developed. It is well to consider the derivations of words used in mathematics.

12 O'Brien, loc. cit.
In order to determine the degree of comprehension of a problem, it is good practice to have a student re-state it in his own words.  

Some teachers have found it helpful to eliminate the use of all numbers from the problem and have the student sketch the procedure for solving it.

One can also ask such questions as:

What are you asked to find?
What facts are given?
What new facts can you discover from what is given?
What computations, in what order, are called for?
What is a reasonable answer? How do you know?

Sometimes a simple problem is obscured by large numbers, or mixed numbers and fractions. If these are replaced by simpler numbers, the situation becomes clear even though the substituted numbers are unrealistic. The student should be encouraged to use such devices.

As with any "foreign" language, difficulty may be expected when the student is asked to translate from English into "Algebra". Since it is common experience that it is harder to translate into a foreign language than from it, we should give more practice translating from algebra into words before the reverse procedure is attempted.


17. O'Brien, loc. cit.
Post-solution analysis is a valuable experience that is widely neglected.\textsuperscript{18} Once a solution has been reached, the student should be encouraged to look back over the way he has come. Looking back on the solution, the students may be led to see that, while there are innumerable problems, many of the problems have the same relationship among the variables. The student should try to see how a specific solution can be generalized and applied to other problems, how it can be changed to create new problems, and how we go about finding general principles.\textsuperscript{19}

"The greatest single ally we can have in teaching problems is interest on the part of the student. Interest might almost be put ahead of intelligence..."\textsuperscript{20}

How can interest be created and maintained?

First of all, we must have good teachers who know what they are supposed to teach. Guy M. Wilson\textsuperscript{21} indicates that most people teaching arithmetic do not themselves realize that the subject they are presenting should be easier to master than either reading or spelling. In arithmetic, children must learn only 681 basic facts. In spelling there are 2,000 justifiable words for the

\begin{itemize}
\item \textsuperscript{18} Kinney, loc. cit.
\item \textsuperscript{19} Stephen S. Willoughby, "Discovery," The Mathematics Teacher, LVI (January, 1963), 22-25.
\item \textsuperscript{20} O'Brien, loc. cit.
\item \textsuperscript{21} Wilson, loc. cit.
\end{itemize}
grades. In reading there are 3,000 basic words, with indefinite extension through dictionary usage.

The mathematics teacher must have enough depth in his field, at all grade levels, so that he can guide and encourage budding geniuses like the seventh grader who, with a few helpful suggestions from his teacher, was able to set up a system for computing any root of a number. 22

The teacher must also be alert enough to head off a potential revolt in the classroom. Ethel Turner 23 describes a situation where such a mutiny was redirected into very desirable channels. One of the students brought to class a copy of the "Penny" comicstrip. The main character (Penny) was shown talking on the telephone and raving on and on about the horrors of algebra. The teacher (after a few quavering moments of doubt) suggested that the class have a contest (with a money prize) to see who could write the best letter answering Penny's protests. It was reported that classes perked up and "lessons brought questions and comments from many previously silent pupils."

22 David Shapiro, "How I Came across the Extraction of N-th Roots," The Mathematics Teacher, LII (March, 1959), 180-83.

23 Turner, loc. cit.
Another way to maintain interest is to use practical applications. But such material must be within the reasonable comprehension of the student—otherwise more are confused than stimulated.

Also, in presenting practical applications to the class—even though the problem is presented in the field from which it comes—the instructor must point out the mathematics involved. If a practical application is to be a challenge to the mathematics student, the mathematics must stand out as such.

Too often practical applications are emphasized in the teaching and they are used to motivate interest, yet the tests cover only mechanical problems. If practical applications are not used in the testing program, it will be found that the students will be much less concerned about them than if they are included.

Even those materials which are presented year after year must be presented in a manner that seems new and interesting. Suggested interest-rousing devices include a homemade slide rule for addition (even the best mathematicians use calculators to do tedious calculations), use of Napier's bones for multiplication,

work with numbers to other bases, and problems based on
newspaper advertisements. 25

At a higher grade level the instructor might introduce
the principle of iteration as it applies to square root
and cube root calculations. He might even introduce simple
computer-type problems which can be solved longhand by
iteration, and at the same time permit the student to be-
come somewhat familiar with how a computer works. 26

Many students attend school all year long and hear
nothing of mathematics outside the mathematics classroom.
More interest can be developed by establishing mathematics
clubs, mathematics newspapers or bulletin boards, mathematics
contests, science fair projects, and the like, which operate
outside the classroom. 27

The emotionally disturbed child must be recognized,
and work with him must be on an individual basis. Here
the first requisite is that the teacher have a firm belief
that the student can succeed. This belief is in some way
transferred to the student and is an indispensable aid. 28

25·Margaret F. Willerding, "Stimulating Interest in
Junior High Mathematics," The Mathematics Teacher, LII
(March, 1959), 197-201.

26·Francis Scheid, "Feed It Back," The Mathematics
Teacher, LII (April, 1959), 226-29.

27·Louis Grant Brandes, "Stimulating an Interest in
Mathematics," National Association of Secondary Schools'

28·Tulock, loc. cit.
The teaching must begin at a level that assures success for the student, so that he may gain confidence as the sessions progress. Once this confidence is established, "rapid and permanent progress" may take place. Such rehabilitated students are sometimes so impressed with the power of mathematics that they go on to become mathematics teachers themselves.
CHAPTER IV

SUMMARY

The difficulties encountered in teaching problem-solving in the high school center mainly on misconceptions developed in the grades. The job of the high school teacher might be eased by providing the grade school teacher with more background in mathematics.

In the meantime, the high school teacher must try to re-teach the subject in such a way that these misconceptions are corrected. In order to do this, a high level of interest must be developed and maintained.

Choosing problems from the students' environment and cultivating mathematics outside the classroom are just two ways of gaining interest. Since, to the student, the material on the tests is the important part of the course, tests must be made to conform to classroom problems.

Mathematics "language" should be carefully developed. "Patterns" for certain types of problems may be set up and the student encouraged to follow them. Interrelations between various problems should be pointed out, or better still, discovered by the student.
The suggestions contained in this paper will not make the teaching of problem-solving easy, but they should present a challenge which will make it much more interesting.


David Shapiro, "How I Came across the Extraction of N-th Roots," The Mathematics Teacher, LII (March, 1959), 180-183.


