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Meaningful Mathematics

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MEANINGFUL MATHEMATICS

(TITLE)

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INTRODUCTION

After twenty-five years of making a living first as a metallurgist in industry, then as the owner and operator of a small corporation, I am presented with the opportunity of choosing a new pattern of living. The extreme pressures of financial responsibilities were relieved. It was no longer mandatory for me to be active in the operation of the corporation. The promise of personal rewards, the varied personal contacts and the challenges of school teaching led me to this profession.

I entered the profession with some definite ideas about the qualifying marks of a good teacher and the type of subject matter most needed by our students. From earlier experiences as a student and as the father of students, I had formed conclusions as to what in education seemed out of phase.

This paper is the result of the combination of ideas formed before deciding to become a teacher and from experiences and readings following that decision. My interests have been chiefly in the group of physical sciences and mathematics.

It had occurred to me that our public schools weren't keeping abreast of the times. It seemed that my youngsters were just coasting after about the fourth grade. It did not appear that the appropriate subject matter was being offered -- especially in the junior high school. My readings and my discussions with people in the teaching field have been following these same lines. Does subject matter need to be changed? Can pupils learn more than is being asked of them? Is more understanding really being stressed, especially in mathematics? Is it possible to make major changes in this field? What does a really good teacher do? The answers to many of these questions make up the greater part of this paper.

CHAPTER I

MEANINGFUL MATHEMATICS

A transfer of the knowledge of mathematics has been a prime aim of the educational systems throughout history. During the past fifty years, life patterns in this country, and in the rest of the world, have been completely altered by scientific developments. Many of these developments were made possible, at least in part, because of companion advances in mathematics. In the future, indications are for a greater need of the understanding of mathematical principles. The role of the teacher will be an important one.

Publicly expressed opinions of many people in the field indicated that perhaps all was not well in the instruction of students in mathematics. It seemed that changes were necessary to provide for the growing subject matter and the increased importance of mathematics in the development of all things. Various groups such as the School Mathematics Study Group (SMSG), the Commission of Mathematics of the College Entrance Examination Board (CEEB), University of Illinois Committee on School Mathematics (UICSM), and kindred groups have been working for several years to ascertain what changes to recommend. Some of the changes that were deemed necessary by these groups were:

1. Stress understanding rather than manipulation;
2. Include set language and concepts;
3. Cultivate an understanding of, and an appreciation for the structure of mathematics;
4. Teach a more refined concept of relations and function.

Other changes of a more specific nature were recommended, relating to the teaching of algebra and geometry, which will not be discussed here.

The federal government, feeling that mathematics instruction in our schools was one of national concern, has worked with these mathematics study groups. That recommendations of these various groups agree with the analysis of the United States Office of Education, can be evidenced in the statement released by Kenneth E. Brown of that office.

> The new uses of mathematics require less manipulation of formulae and equations but a greater understanding of the structure of mathematics and mathematical systems. The role of mathematics is not only to grind out answers to engineering problems but to produce mathematical models that forecast the outcome of the social trends and even the behavior changes of the group.[1]

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[1]Kenneth E. Brown, "Mathematics Today," Mathematics Teacher, LIV (February, 1961), p. 85.

Volumes have been written on the undesirable lack of retention of subject matter by students. If the studied concept is not related to the pupils perception of concrete things, the retention of these concepts will be greatly reduced. A language learned in the classroom or a science studied but not applied or related to everyday experiences is usually soon forgotten. Although there is almost complete agreement that the need to increase retention of knowledge exists, it is in the solution of the problem that we do not find accord. Other than repetition and practice, some of the paths suggested for leading the student to the desired goal of increased retention of knowledge, were: more student participation in the classrooms, stronger motivation, more interest and increased application of principles. These latter suggestions can be summarized into making the subject more meaningful. Most of the teachers in our high schools and junior high schools prescribe this as the remedy with some disagreement as to the amount of routine practice still needed by the students. Miss Alta Meier, the writer's supervisor during student teaching at Pana, Illinois, High School, felt that she could accomplish much more in increased understanding and retention if she could relate the principles being studied to something more closely associated with her students. Miss Meier, highly regarded by her colleagues, has continued her education while teaching mathematics for the past twenty years. She has had experience with the traditional methods and with the transition to modern mathematics and should be considered a competent judge of classroom performance. Miss Meier, however, must also be counted among those advocating practice, as a means to impress the import of the principle and its usage, upon the minds of the pupils.

It would seem that the ideal course content in mathematics would be such that it contained the necessary concepts leading to an understanding of the structures of mathematics, and that these concepts be introduced in such a way as to be interesting and related to some known concrete examples. The problem of lack of appreciation and understanding must have existed in other countries as well. British educators were aware that changes were needed in course content at least thirty years ago.

> The joys of pure mathematics, the study of algebraic form, the appreciation of an elegant method are for the pure mathematician and not the average boy.[2]

The new approach to modern mathematics in high school algebra textbooks is beginning to remove some of the barriers that divided mathematical concepts from the realistic world of the student. At present, mathematics is being revealed to our students as a man-made logical science, no longer semi-divine, but often in a profound abstract manner.

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[2]British Report, The Teaching of Algebra, New York: Alfred A. Knopf, p. 7.

The exposition of the relationship of these abstract generalizations to the concrete physical world, familiar to the student, would remove another formidable barrier to the understanding of the concept by the student.

Specifically, if a course of study at the junior high level was designed to include the usual elementary concepts of algebra, geometry and trigonometry, combined with the basic principles of mechanics, gravitation, electricity and magnetism, and perhaps some nuclear physics, it would constitute a foundation for further study in a specialized area of mathematics or physics for the better student and still be a basic terminal study for the lesser endowed students.

In some of the elementary schools, mathematics is being presented in a manner best considered to stress structure and concepts. Even at this early age, youngsters are aware of the relationship and comparison of things outside their formal education. The need to describe size and shape and to locate positions arises every day. It is in these things that the generalizations and relationships in mathematics can best be introduced. In this way, the concept would precede the manipulation and the discovery of the relationship would give more meaning to the concept.

It has been recognized for some time that we have not been providing the proper program of study for our children. As early as 1902, this observation was published:

> I understand that serious difficulties arise with children from nine to twelve years of age, who are no longer contented with the simple concrete methods of earlier years, and who, nevertheless, are unable to appreciate the abstract methods of the later years.[3]

However, more than fifty years later the situation still seemed to exist.

> Very few people are aware that children, ordinary children, can deal with mathematical situations of a much more elaborate nature than those with which they are usually presented.[4]

There are concerned parents who recognize that this generation of children lose interest and enthusiasm for school at about the same period in their schooling as did their parents. At this point, it would seem that it would be possible to introduce the type of course mentioned by eliminating the repetitious subject matter taught during this period. If any doubt exists that the present subject matter is repetitious, an examination of the text-books of Grade 5 through Grade 8 in an average public school will dispel that doubt. This is true in mathematics as well as other subjects. The constant repetition does little to increase understanding, and what is gained in skill is usually lost in the students dulled quest for knowledge.

There are other factors that indicate success may be possible by

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[3]Eliakim H. Moore, On the Foundation of Mathematics, (First Yearbook of National Council of Teachers of Mathematics, Washington D.C.), p. 45.

[4]C. Gattegno, "Observation of the Teaching of Mathematics in the United States," Mathematics Teacher, LI (February, 1958), p. 194.

the use of a combined course to promote understanding. If the life experience of today's teenager was compared with that of the youth of a generation ago, we would surely find that the experience range of the modern youth is much greater than his ancestor's experience range, and the gap increases day by day. A comparison of their behavior patterns will show that the modern youth is allowed much more freedom of thought and is not as closely dominated by the adult world as were his parents. If at any time in history there existed a "why" generation, it is now. Our advances in science have been so rapid that common sense no longer can be used as a universal guide.

Of course, providing the awareness of the relationships between the concept and the student's known world will be the major step if we are to inject more meaning into our presentations of mathematics. The teacher and the textbook must furnish this one missing ingredient to our present educational picture. After reading the book, Mathematics and the Physical World, by Morris Kline, it is easier to visualize a science-invaded mathematics class with active group response and understanding. Being a member of a mathematics class led by Dr. Lawrence Ringenberg or a Physics class led by Dr. Scott Smith, both at Eastern Illinois University, helps solidify the belief that a combination of physics and mathematics, when properly presented, would be the ideal way to promote learning and understanding in mathematics. Dr. Karl Menger, who has published several articles dealing with the need for an increase in meaning in mathematics, taught beginning college algebra in much the same manner as we are suggesting. Dr. Menger was doing this thirty years ago at Notre Dame University. Morris Kline, in an effort to emphasize the need for the physical aspect used this example:

> An attractive young lady weighing 110 pounds and unable to swim is sitting on the edge of a diving board 10 feet above the water and sunning herself. She is pushed off the board. How long does it take her to strike the water? Insofar as the mathematics is concerned the attractive lady is a mass and her mass may, for the purpose of the problem, be regarded as concentrated at one point. She is, in other words, a point mass. The fact that she is pushed off the board means to the mathematician that she starts her fall with zero velocity. Since she falls a rather short distance, the mathematician will disregard the resistance of the air. Moreover, when air resistance is ignored, all objects fall at the same rate. The young lady's weight is therefore immaterial. But objects falling straight downward under the pull of gravity cover a distance d equal to $16t^2$ feet in t seconds. Hence the mathematician will substitute 10 for d in the formula and find the corresponding value of t. The entire physical problem is converted into a problem in algebra. Mathematics unquestionably fails to notice some of the beauty in this world in its proclivity for idealizing, yet such are its ways.[5]

It has required a major effort on the part of mathematics groups, teachers, universities and publishers, to make an inroad into the traditional mathematics approach. Modern mathematics programs are slowly proving that they have merit, and are being introduced in many schools.

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[5] Kline, Morris, Mathematics and the Physical World, Thomas Y. Crowell Co., New York, 1959, pp. 24-25.

To break established patterns of thought and methods means that the inertia of status-quo must be overcome. Because of the intense preparation required of the leaders in the educational fields, it is often difficult for these people to maintain a perspective compatible with the attitudes and aims of the groups they influence. The feeling is prevalent within the rank and file of school teachers that it is futile to attempt to bring about any change in educational methods regardless of complete unanimity within the group that the change is needed. It is for this reason that many apparently sound and commendable methods can never be evaluated.

One concludes that in teaching, like many other professions, there are numerous solutions to the many problems encountered. The teacher's training courses cannot draw an unwavering line to success. The educational departments at our universities can show, at best, only the general path to follow. It becomes the duty of the teacher to select and present those concepts that will generate the most complete understanding of their particular discipline.

In mathematics, therefore, if the most meaning and understanding can be obtained by fitting it into its natural surroundings, why keep it separated?

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