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A BRIEF HISTORY OF SOME COMMON AMERICAN

UNITS OF LENGTH AND WEIGHT

(TITLE)

BY

Melvin M. Williams

PLAN B PAPER

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
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I HEREBY RECOMMEND THIS PLAN B PAPER BE ACCEPTED AS
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INTRODUCTION

The purpose of this paper is to compile material that can be used to supplement a unit on measurement for Junior High School students. This paper contains only a representative sampling of available material.

Although the writer has read many books dealing with the history of measurement, he has not attempted to give credit through footnotes for all ideas outlined in this paper. To have made such an attempt would have resulted oftentimes in a complex maze of names and references at the bottom of a page. The reader should refer to the Bibliography for the list of sources from which the writer has obtained his information. A raised numeral in the text refers the reader to the footnote and to the complete reference in the Bibliography.

Apparently there has never been a time when man has not struggled with the problems of measurement. Much of the time similar struggles have occurred simultaneously in different countries. Yet the system of measures that has been developed in one country often has had little or no relation to that of another. For example, a Polish farmer would probably measure his grain harvest by the korzec while an American farmer would measure his by the bushel.¹⁴

¹⁴Myron F. Roskopf et al, Modern Mathematics, Book I, p. 158.

Not only have the systems used in different countries been unrelated, but even the units found within a single system. Little relation has existed because the units apparently were invented and used under special conditions. Seemingly they were chosen with little thought as to how they could be incorporated into a table of measures. Let us see how these systems probably arose.

To the best of our knowledge, early man had no measuring instruments. When he wished to measure an object, he probably used anything that was convenient. If he wished to make a club the same length as his neighbor's, he probably used his neighbor's club to measure with. However in most cases, the objects that were nearest were the parts of his body. If he wished to measure the length of something he could handle, he probably counted the number of times he could place his finger side by side along the object. For something longer, he might have counted the number of hand-widths in its length. If the object had been even longer, he might have counted the number of forearm-lengths. Since its length would probably not have been a whole number of forearm lengths, he might have continued the measurement in hand-widths and finally in finger-widths.

To measure distances on the ground, he probably found it most convenient to use the part of his body that touched the ground--his foot. For slightly longer distances, he probably used the length of his step. These lengths then became his units of measure.

All of early man's needs to determine length were not met by the kind of units mentioned above. If he wanted to tell how

far away a distant object was, he probably told the number of days it would take him to walk there.

If four finger-widths equaled one hand-width, it did not necessarily follow that four hand-widths would equal one forearm-length or one foot-length, or the length of one step. It was even more ridiculous to try to find a relationship between a body measurement and a time measurement.

Units for measuring length were not derived systematically. They merely grew out of everyday experiences. Generally they were associated with the human body. In spite of this lack of system, many of these earliest units are still in use today. A knowledge of their origin gives one a better understanding of our systems and of the advantages of a planned system.

The writer is indebted to Dr. David Davis for his time, inspiration, and constructive criticism.

CHAPTER I

UNITS OF LENGTH DERIVED FROM THE HANDS AND ARMS

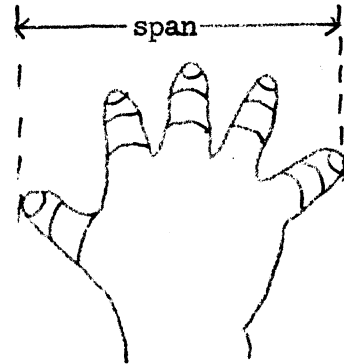
The smallest unit that originated from measuring with a part of the hand was called the digit or the breadth. It was the width of either the first or the middle finger.

The next larger unit was called hand or palm. It was the width across the open hand at the base of the fingers. When one measures the width of his palm and his digit, he finds that one palm is approximately four and two thirds digits--a relationship that is not convenient!

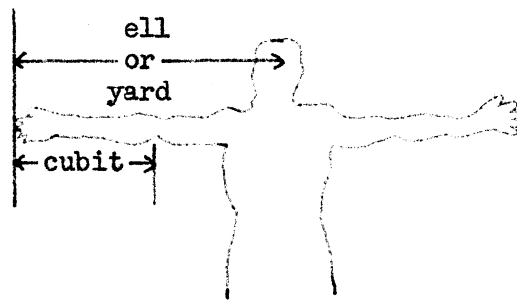
digits



The span was the distance between the tips of the thumb and the little finger when the hand was spread. If one measures his span, he finds it is approximately two and one-half palms--another relationship that is not convenient!



Cubit comes from cubitus which is the Latin word for elbow. A cubit was the length from the point of the elbow to the tip of the outstretched middle finger. It was approximately two spans,

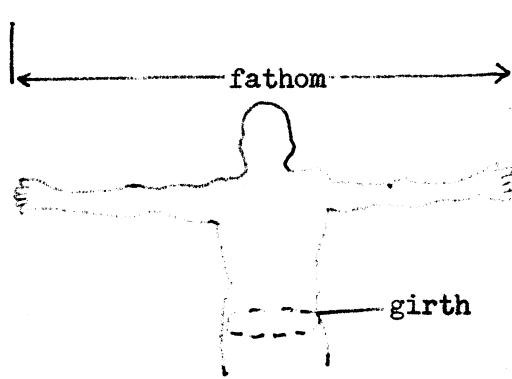


six palms, or twenty four digits. It varied from about seventeen and eight-tenths inches to about twenty-six and eight-tenths inches. There were two particular cubits of outstanding importance --the Royal Egyptian cubit of twenty and sixty-two hundredths inches and the Olympic cubit of eighteen and twenty-four hundredths inches.

The yard originated from two units of measure which were about the same length. The ell was one. In some countries it was the distance from the middle of the chest to the tip of the middle finger when the arm was stretched out horizontally to the side. In other countries, the ell was the distance from the end of a man's nose to the end of his thumb. In southern Europe it was a double cubit. Some think ell came from elbow even though it became a much longer unit than the cubit. It is possible that certain ells were related to the length of an arrow, since an arrow is about as long as the distance from the center of the chest to the fingertips when the arm is outstretched.

The other unit which gave birth to the yard was the girth. It was the distance around a man's waist. Some say girth, girdle, and yard may have come from the same root. This belief is supported since northern Europeans, especially the Saxons, defined

their yard as a girth. This Saxon yard spread to the English and then to us.



The largest unit which originated from the use of the hands and arms was the distance between fingertips when both arms were outstretched. Since the length of both arms was required to embrace someone, the Saxons called this unit the fathom. Fathom was their word for embrace. It was either approximately or exactly two ells, depending on the definition of the ell. The fathom was also approximately two girths.

CHAPTER II

OTHER UNITS OF LENGTH

The uncia was the smallest unit that originated from the need to measure distance on the ground. Some people believe the uncia was originally the distance from the top joint to the tip of the longest finger. The Hindus called it finger-part. Others believe the foot originated first and later was divided into unciae (inches). They believe the foot originated first because uncia was the Latin word for twelfth and an inch is a twelfth of a foot. Still others think that the uncia originated as the breadth of the thumb. Of course it is possible that it originated in all three ways, each way originating in a different country or in different sections of the same country.

Apparently the foot became a unit of measure because man's foot was the part of his body that could be used most conveniently to measure distances on the ground. We do not know when or where it was first used, but archeologists and historians have found that it ranged in size from nine and three-fourths inches to nineteen inches. The reason for this variance was probably that man's foot became smaller when he took to fancier sandals. Nevertheless, by the time of the Greeks, the foot had decreased to approximately twelve inches. The foot was such a convenient unit that it came to be more commonly used than the cubit. The Greek foot was about

two-thirds of a cubit--another relationship that is inconvenient.

The twelve-inch foot that we use today is common only in English speaking countries. In other countries it ranges between eleven inches and fourteen inches. There is a legend that the United States standard foot was originally the length of the foot of Charlemagne, who was a tall man.

The next larger unit was the pace. It was a double-step, approximately five of our feet.

The rod, was simply an oxgoad--a stick that was carried by the English to goad oxen when plowing. The oxgoad (oxrod) was used by plowmen to measure the space between the two outside furrows. The rod was also called a perch. Perch came from the Latin word pertica which meant pole or rod. Obviously, the origin of the rod was connected with farming.

Greater distances were often described as a certain number of arrow flights. An arrow flight was the distance an arrow could be shot.

Another unit of similar length was the furlong. Furlong was an abbreviation of furrow long, which was the length of a furrow oxen could plow without stopping to rest. The furlong was eventually set equal to two hundred twenty yards.

League came from the German word lugen. The league was supposed to be the distance a person could see with his naked eye or approximately three miles.

Our mile originated with the Romans. As Roman soldiers marched along the highways they counted paces and placed markers at the end of each thousand-pace distance. They named this

distance millia passuum, or mille for short. When the use of mille spread to Europe, the English renamed it mile. Since the Roman pace was about five feet, the Roman mile was about five thousand feet.

CHAPTER III

ATTEMPTS TO STANDARDIZE UNITS OF LENGTH

We have seen how man used parts of his body as measuring sticks and how, for example, there was no noticeable relationship between the palm and the pace or the span and the step. Obviously, one person's hand or foot was not the same size as another's. There was no standard palm or foot. This did not matter as long as each person worked alone and measured for his own needs. But, it was not feasible to use the handbreadth of a large person in buying a string of beads and the handbreadth of a small person in selling them. It became necessary to agree upon common measures so that goods could be exchanged conveniently.

Man also realized he could measure more accurately if he had a stick the length of each unit. But this posed a problem. It was difficult to carry all these sticks and he often needed several of them to measure one length. Finally, he seemed to realize that a single cubit stick would solve his problem. He simply marked off the span, the palm, and the digit on this one stick.

The early Babylonians, Egyptians, and Hebrews, began to use the same size cubit. Once the cubit was standardized, other units were standardized too. In time, smaller units became closely related to larger units, and tables of measures appeared. Instead of being approximately one-fourth palm, the digit was set equal to

one-fourth palm. As man associated the smaller units with the larger ones, he found the following relations convenient.

3 feet equal 1 ell
5 feet equal 1 pace
6 feet equal 1 fathom
 $5\frac{1}{2}$ ells equal 1 rod
40 rods equal 1 furlong
8 furlongs equal 1 mile

Trade guilds in the same city often had different standards, and rulers were marked with the names of the cities in which they were used. As medieval fairs began and trade was increased between communities, a new need for common standards was discovered. Consequently, it became customary for the person who was the leader of the group, possibly the king, to require that his cubit or his ell be the standard unit for the entire empire. As one studies the history of measurement, he finds that most standards of weights and measures have been fixed by the rulers of countries or by the leaders of tribes, rather than by houses of congress or parliaments.⁷

From time to time, measures of length have been used which were never standardized or included in any table. On the other hand, some measures that were standardized were later dropped. Ancient weights and measures were invented and used to meet special requirements. When they were no longer needed, they disappeared. For example, when the yard became standardized, it was no longer necessary to measure cloth by wrapping it lengthwise around the forearm. This led to the disappearance of the ell.

⁷Jeanne Bendick, How Much and How Many, p. 21.

The Romans were quite active in standardizing units of measure. They borrowed the foot from the Greeks, revised it slightly, and required that this foot be used in all parts of their vast empire. All of their standards spread throughout Europe, Western Asia, England, and Africa as they conquered these places. These standards were kept in a temple in Rome, and became the foundation of the local systems.

When the Roman Empire fell, the standards were lost. Measures began to differ in different parts of the old empire until almost every town and every guild in the town had its own system of weights and measures. Even in the other parts of the empire, the standards became so inaccurate and mixed up that many of them dropped out of use.

The Hindus set the inch equal to the length of three barleycorns placed end to end, or equal to the breadth of eight barleycorns laid side by side. Again we find a unit that was directly connected with farming.

For centuries attempts had been made to make the units of length the same throughout England. The foot which the Romans borrowed from the Greeks was passed on to the British. In Britain the foot merged with the Anglo-Saxon measures which contained the fathom. As the English stabilized their units of length, they found the five thousand foot mile was inconvenient because it was a little more than seven and one-half furlongs. They decided to make it eight furlongs, so the thousand-pace mile became about 1,050 paces. This is the reason the mile is now equal to 1,760 yards and 5,280 feet.

In the early part of the twelfth century we find that King Henry I started with the yard in attempting to standardize the measures of his time. He decreed that the yard should be the distance from the end of his nose to the end of the thumb of his outstretched hand. This is the length of the yard we use.

At the same time in Scotland, King David gave directions for determining the inch. These directions stated that the inch was to be the average width of "the thowmys of iij men, that is to say an mekill man and a man of messurabil statur and of a lytell man. The thoums are to be messurit at the rut of the nayll."⁵

Later in the twelfth century King Edward I decreed that a standard yard be made from an iron bar and declared that the foot was to be one-third of this standard yard.

In the next century things had become so confused that standards had to be set up for various measures. The foot measures throughout England were made from the iron bar that was kept in Saint Paul's Church. The ell was made from the iron ell in the King's Palace. Rather precise copies of these standard measures were made and sent to various cities and market places throughout England.

In the fourteenth century, King Edward II set up this table of standard units.

1 inch equals 3 barleycorns (round and dry) placed end
to end
1 foot equals 12 inches
1 yard equals 36 inches or 3 feet
1 perch (or rod) equals $5\frac{1}{2}$ yards

⁵American Council on Education, The Story of Weights and Measures, p. 11.

The barleycorn inch mentioned in the above table is still used in determining shoe sizes. When King Edward II determined the barleycorn inch, the longest normal foot was assumed to be thirteen inches.⁹ A shoe that fitted such a foot was said to be size thirteen. Smaller sizes were graded down from thirteen, each by a third of an inch--the length of a barleycorn.

In the fifteenth century, Parliament again attempted to standardize units of length and started with the inch as King Edward had done the century before. Parliament required that to determine an inch, three barleycorns be placed end to end, be round and dry, and be taken from the middle of the ear.

As late as the sixteenth century the Germans had no standard measuring rods. In an old German surveying book the following plan was given for finding the lengths of the rod and the foot.

"Stand at the door of a church on a Sunday, and bid sixteen men to stop, tall ones and small ones, as they happen to pass out when the service is finished; then make them put their left feet one behind the other and the length thus obtained shall be a right and lawful rood (rod) to measure and survey land with, and the sixteenth part of it shall be a right and lawful foot."²

Since most of our early settlers came from England, most of our units of measure are those that were used in England. However, most of our common units of measure have had only minor changes since the time of the Saxons. For example, the yard used in those days was different from ours by approximately one-

⁹Compton's Pictured Encyclopedia, Vol. XIV, p. 178.

²How Long is a Rod?

hundredth of an inch.⁷ The story of the history of our units is mainly the story of the history of English measures.

Even though England and other parts of the British Isles had long attempted to standardize units of measure, the colonists came to our shores with units that were not consistent. As time went by these units revealed even less uniformity. Soon each colony had its own standard yard. This condition continued until approximately fifty years after the Revolution. Then Congress adopted the thirty-six-inch yard that the British were using and distributed uniform copies of it to the custom houses. For more than half a century the custodians of the standard measures of the United States tried to keep the yard equal to the one used by the British.

In 1893 a standard of length was chosen that was different from the British yard. At that time the United States joined most of the world in adopting the meter. It was the distance between two scratches on a non-rusting bar made of a mixture of platinum and iridium. It is kept in containers in a safe in a vault under one of the buildings of the International Bureau of Weights and Measures near Paris. Three keys are needed to unlock the vault, the safe, and the containers. These keys are kept by members of the international committee who live in different countries. This committee meets once every six years, opens the vault, and examines the standards. A copy of this bar is kept at Washington, D. C., and has been the basis for all of our measures of length. Congress

⁷Bendick, loc. cit.

passed a law making the United States yard equal to $3600/3937$ of this meter.

In 1960 a more precise standard for the meter was adopted. It is now defined as a multiple (1,650,763.73) of the wave length in a vacuum of the orange light of an atom of the gas Krypton 86. Measurements can be made to the nearest one-millionth of an inch with this standard. Since this standard can be produced in a well equipped scientific laboratory, it is unnecessary for scientists to visit the laboratory of the Bureau of Standards. This laboratory is the greatest research and testing laboratory in the country and is in charge of all of our standards.

CHAPTER IV

THE BALANCE AND EARLY WEIGHTS

When arrowheads were traded for skins there was no need to measure. The trader counted so many arrowheads for a small skin and so many for a larger skin. But when he was selling and buying things that could not be counted, like wine and grain, he thought of the weight of the one as well as the weight of the other. But there was a time when he did not have a weighing device.

The first weighing machine was probably invented by the Egyptians somewhat over five thousand years ago. It was called a balance. It consisted merely of a stick suspended in the middle by a rope. The objects to be weighed were suspended from each end of the stick by other ropes.

This type of balance was soon improved to include a pan suspended from each end in which the things to be weighed could be placed. Along with this improvement came the idea that seeds or stones could be used as weights in one pan to balance the object being weighed in the other. If the stick remained level, the objects were equal in weight. If one end of it dropped, the object at that end was the heavier of the two. This kind of balance was used to measure both liquids and dry materials. It was just as common to weigh wine as it was to weigh flour or grain. Of course, if wine was weighed, a lighter stone was used. If a

precious metal was to be weighed, the weights had to be very light, so seeds or grains were used.

As early as 3,000 B. C., the Egyptians chipped small weights out of stone to represent certain amounts of some particular commodity. Later, small pieces of metal were used with each piece of metal being made equal to a certain number of seeds or kernels of grain. Archeologists have found some very interesting weights from these early civilizations.

At first, the weights were the same only in a single shop or in the village market place. If a man wished to weigh his grain at home, he would have to have a stone equal in weight to the one in the market place. As travel and trade increased, it became important for entire countries to use the same units of weight. These were developed in much the same manner as were the standard units of length.

CHAPTER V

UNITS OF WEIGHT AND ATTEMPTS TO STANDARDIZE THEM

The unit of weight called the grain originated from using grains of wheat or barley to weigh small and light things. It is still the basic and smallest unit of weight that we use for weighing drugs and precious metals. It has often been used to determine the value of money. A piece of money was valued according to the number of grains that it weighed.

Rati-seeds, carob seeds, and seeds of the wild licorice were also used to determine the value of money. A coin was said to be so many carats, meaning that it was equal in weight to a certain number of carob seeds.

We still weigh diamonds by the carat. The carat also used to indicate how fine the gold is in a particular thing. The carob seeds were used because they, like the rati-seed, varied so little in size. Some say that carat came from an Arabic word meaning a kind of bean--the bean from the Abyssinian tree.¹⁰

The pennyweight was simply the weight of a silver penny that was coined in England in the Middle Ages. It is still used in our Troy system and is equal to twenty-four grains. We find several accounts of attempts to standardize the pennyweight. In 1266, King Henry III of England decreed that

¹⁰Susan Cunnington, The Story of Arithmetic, p. 68.

"...an English penny, called a sterling, round and without any clipping, shall weigh thirty-two wheat corns in the midst of the ear, and twenty pence do make an ounce, and twelve ounces one pound, and eight pounds do make a gallon of wine, and eight gallons of wine do make a London bushel."¹⁸

The ounce originated as the twelfth part of the Roman pound. The words ounce and inch both came from the Latin word uncia, which meant the twelfth part.

Pound seems to have come from the Greek word meaning the weight or to have some weight. Very small and light things were not thought of as having weight as were larger and heavier things. Larger and heavier things were said to be as heavy as pounds.

The pound became a very important measure during Roman times. The Romans spread its use throughout their entire empire. They set it equal to 7,680 grains and called it libra. It was from libra that we got the abbreviation lb.

Since the pound was first invented, there has been the idea that there should be two different kinds of pounds--one for weighing light things of great value and one for weighing heavier things of lesser value. This idea has passed on to us. Even today a pound of iron and a pound of gold do not weigh the same, because they are measured in two different kinds of pounds.

The pound with which we measure highly valuable things is the Troy pound. It is supposed to have been brought into England in the fourteenth or fifteenth century by the French from their market place in Troyes. At that time it was equal to 5,760 grains. It was also defined as 5,760 grains by King Henry III in 1266, was

¹⁸Harry Grove Wheat, How to Teach Arithmetic, p. 384.

brought to America in the colonial days, and is still the same in our Troy tables. This Troy pound is 0.8229 of the Avoirdupois pound in present day use.

The Avoirdupois pound is the pound with which we weigh less valuable and heavier things. It also was brought into England by the French and into America by the English. The French word Avoirdupois meant goods of weight. In weighing heavier goods it was thought the pound should be heavier, so the French used a seven thousand grain pound to weigh their less valuable goods. They divided it into sixteen ounces.

In 1303, Edward I of England attempted to standardize the Avoirdupois pound by adding four ounces to the old twelve-ounce Troy pound. Thus, the Avoirdupois pound became one and one-third times as heavy as the Troy pound and equal to the old Roman libra of 7,680 grains.

King Henry VII also attempted to standardize units of weight by making a decree similar to that of Henry III. Then in 1532, it was decreed that such goods of weight as beef, woolens, etc., should "...be sold by weight called Haverdupois."¹⁸ In the time of Henry VIII, the Avoirdupois pound of 7,000 grains came into use. This Avoirdupois pound was standardized in our country in 1893 at the same time that the yard was standardized. We still use the Avoirdupois pound for weighing all commodities except drugs, precious stones, and metals.

The stone was probably the first weight. It varied from as little as four pounds to as much as twenty-six pounds. It is no

¹⁸Ibid.

longer used in the United States, but is still a common unit of weight in England. The English set it equal to fourteen pounds.

The hundredweight, as its name implies, is one hundred pounds. However, there is a long hundredweight of one hundred twelve pounds that is often used commercially. Since the hundredweight of one hundred pounds was equal to seven and one-seventh stones, Queen Elizabeth added twelve pounds to the hundredweight to make it eight stones. Even today, the hundredweight in England is one hundred twelve pounds.

The ton is thought to have originated in northern Europe. Some say it started as a tun, a huge cask used to hold wine. Others say it started as a chaldron, a wheat measure that held about thirty-two bushels. Nevertheless, it was set equal to twenty hundredweights. Twenty hundredweights equal two thousand pounds, or the short ton in use today. The short ton is in common use in the United States, Canada, and South Africa.

Twenty long hundredweights equal 2,240 pounds which we call the long ton, and which is the ton used most commonly by the English. The long ton is used frequently in commerce in the United States.

In some states, the statutory ton of certain commodities is defined as a definite number of cubic feet or as a certain number of bushels of fixed weight.

Congress has the constitutional right to fix the standards of weights and measures, but it usually has left this task to the states. Therefore, even today, there is considerable variation in standards among the various states.

SUMMARY

As has been indicated throughout this paper, units of measure were chosen to meet special needs under special conditions. Many times, units originated from parts of the body and from farming. Seemingly, these units were chosen quite carelessly. As a result, the tables of measures in use today are difficult to remember and awkward to use. Man has repeatedly attempted to standardize and improve these tables. He has invented new units and new ways of defining the standards in order to satisfy the need for more precise measurement. This does not mean all of the struggles with measurement are over. Can you think of some changes that might be made in the future?

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