7-1-1928

# Bulletin 101 - Arithmetic Teachers in the Making

E. H. Taylor
*Eastern Illinois University*

# The Teachers College Bulletin

Number 101 July 1, 1928

Eastern Illinois State Teachers College

AT

CHARLESTON

# ARITHMETIC TEACHERS IN THE MAKING

BY

E. H. TAYLOR

Head of the Department of Mathematics

(Read at the Annual Meeting of the
American Association of Teachers Colleges,
Boston, February 24, 1928)

# The Teachers College Bulletin

PUBLISHED QUARTERLY BY THE EASTERN ILLINOIS
STATE TEACHERS COLLEGE

Entered March 5, 1902, as second class matter, at the postoffice at Charleston, Ill. Act of Congress, July 16, 1894.

No. 101 CHARLESTON, ILLINOIS July 1, 1928

# ARITHMETIC TEACHERS IN THE MAKING

BY

E. H. TAYLOR

Head of the Department of Mathematics

(Read at the Annual Meeting of the

American Association of Teachers Colleges,

Boston, February 24, 1928)

Eastern Illinois State Teachers College at Charleston

(Printed by authority of the State of Illinois)

# ARITHMETIC TEACHERS IN THE MAKING

by E. H. TAYLOR

*Head of the Department of Mathematics*
*Eastern Illinois State Teachers College*

THE theme of this programme is the progress made in education in the last twenty years. The progress made in the teaching of arithmetic in that period has been significant and fundamental, in methods of teaching, and in the selection and organization of subject matter. There has been a continuous reorganization of subject matter on the basis of learning and use. We have less hesitation in teaching something about 6000 before teaching everything about 6; we teach the simple equation and negative numbers instead of cube root and true discount; we are less likely to give eight times as much practice on 2 x 2 as on 9 x 8 as one text of a few years ago did; and we try harder to show how 9 x 8 is useful in problems the answers to which children want to know. American texts in arithmetic are as well designed for their purpose as any in the world. We are very much alive to the need of keeping the curriculum plastic and susceptible of change to meet the needs of the pupils. The most important problem, the one facing the normal schools and teachers colleges, is to furnish good teachers, masters of the subject and trained to teach. The object of this paper is to exhibit the standards for the preparation of teachers of arithmetic now existing in American teacher-training institutions.

### AMOUNT OF ARITHMETIC
### OFFERED IN NORMAL SCHOOLS AND TEACHERS COLLEGES

A letter requesting a copy of the curriculums offered was sent to every state and city normal school and teachers college listed in the Educational Directory of the United States Bureau of Education for 1927 (Bulletin No. 1, 1927). Copies of the curriculums, in a few cases only letters concerning the courses in arithmetic offered, were obtained from 189 institutions located in 42 states. Two of these institutions do not offer courses for the preparation of elementary teachers and were not included in this study.

A study of the curriculums of 187 of these teacher-training institutions was made to determine the amount and character of the work in arithmetic offered and required. Some of the results of this study are given in the following tables:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| NO. OF COURSES | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | N.C. | UNK. |
| NO. OF SCHOOLS | 8 | 41 | 54 | 24 | 33 | 13 | 2 | 2 | 6 | 4 |

TABLE 1. Number of Courses in Arithmetic Offered in 187 Normal Schools and Teachers Colleges

Table 1 shows that 8 schools offer no course in arithmetic, 41 schools offer 1 course, 54 schools offer 2 courses, and so on; also that 6 schools offer only non-credit courses (N. C.) for students failing to satisfy a test given at entrance; and that the number of courses offered by 4 schools could not be determined from the data at hand (UNK.).

This table, as well as the others that follow, includes as a course in arithmetic any course given wholly or in part to instruction in the subject matter, methods of teaching, or a combination of subject matter and methods of teaching arithmetic. Courses in methods are frequently given by departments of education.

The mode in Table 1 is seen to be 2 courses, and 152 of the 187 schools offer from 1 to 4 courses in arithmetic. The offering of more courses indicates that more differentiation is made to meet the needs of teachers of different grades.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| SEMESTER HOURS | 1-1.9 | 2-2.9 | 3-3.9 | 4-4.9 | 5-5.9 | 6-6.9 | 7-7.9 | 8-8.9 |
| NO. OF SCHOOLS | 2 | 23 | 15 | 16 | 25 | 17 | 2 | 15 |

| | | | | | | |
|---|---|---|---|---|---|---|
| SEMESTER HOURS (*cont.*) | 9-9.9 | 10-10.9 | 11-11.9 | 12-12.9 | 13-13.9 | 14 |
| NO. OF SCHOOLS | 5 | 11 | 17 | 6 | 4 | 7 |

TABLE 2. Number of Semester Hours of Arithmetic Offered

In Table 2 the numbers in the top row represent semester hours. Some institutions give credits in semester hours and some in quarter hours, a quarter being 12 weeks. In this paper all credits have been reduced to semester hours. Table 2 then shows that 2 schools offer from 1 to 1.9 semester hours of arithmetic, 23 schools offer from 2 to 2.9 semester hours, and so on. In a few cases an hour of methods in arithmetic has been counted, although the work in methods in arithmetic is a part of a course in teaching several of the common branches and the time given to arithmetic is probably less than one semester hour. More than one-half of the schools offer from 2 to 6.9 semester hours of arithmetic.

| SEMESTER HOURS | 1-1.9 | 2-2.9 | 3-3.9 | 4-4.9 | 5-5.9 | 6-6.9 | 7-7.9 |
|---|---|---|---|---|---|---|---|
| SUBJECT MATTER | 1 | 20 | 3 | 7 | 5 | 3 | 2 |
| METHODS | 13 | 39 | 13 | 8 | 7 | 8 | 0 |
| SUBJECT MATTER AND METHODS | 3 | 29 | 10 | 14 | 13 | 11 | 2 |

| SEMESTER HOURS (*cont.*) | 8-8.9 | 9-9.9 | 10-10.9 | 11-11.9 | 12-12.9 | 14-14.9 | Totals |
|---|---|---|---|---|---|---|---|
| SUBJECT MATTER | 3 | 1 | 1 | 0 | 0 | 0 | 46 |
| METHODS | 3 | 1 | 3 | 0 | 0 | 0 | 95 |
| SUBJECT MATTER AND METHODS | 9 | 0 | 3 | 15 | 1 | 2 | 112 |

TABLE 3. Contents of Courses. Showing the number of schools giving the different numbers of semester hours to courses dealing primarily with subject matter, with methods, or with subject matter and methods

Using the second column as an example, Table 3 is read as follows: 20 schools offer from 2 to 2.9 semester hours of arithmetic dealing primarily with subject matter; 39 schools offer that number of semester hours in courses dealing primarily with methods of teaching; 29 schools offer that number of semester hours in courses dealing with both subject matter and methods of teaching. This table shows that 46 schools offer subject matter courses, 95 offer methods courses, and 112 offer courses dealing with both subject matter and methods of teaching. An offering of from 2 to 2.9 semester hours is seen to be the mode for each of these three kinds of courses. This means that a quarter's work of 4 hours is a quite general offering in each of these kinds of courses.

In connection with this table it should be stated that 8 schools offer no work of any kind in arithmetic; 6 schools offer only non-credit courses for students who do not pass an examination given at entrance; and 7 schools offer no arithmetic except as a part of a course devoted to methods of teaching several subjects, for example, 2 2/3 semester hours on teaching the common branches. These 7 schools might fairly be counted as offering no arithmetic, which brings the number of such schools up to 15.

### AMOUNT OF ARITHMETIC REQUIRED IN NORMAL SCHOOLS AND TEACHERS COLLEGES

The three tables above show the amount and character of the arithmetic offered. But to know how this affects the teaching of arithmetic in the public schools we must know how much of it is required of students preparing to teach in the elementary schools.

| SEMESTER HOURS REQUIRED | 1-1.9 | 2-2.9 | 3-3.9 | 4-4.9 | 5-5.9 | 6-6.9 | 7-7.9 | 8-8.9 |
|---|---|---|---|---|---|---|---|---|
| RURAL | 0 | 20 | 20 | 6 | 5 | 1 | 0 | 0 |
| KINDERGARTEN—PRIMARY | 8 | 45 | 14 | 20 | 6 | 7 | 0 | 0 |
| INTERMEDIATE | 1 | 31 | 27 | 26 | 11 | 7 | 2 | 1 |
| GRAMMAR | 1 | 27 | 7 | 28 | 11 | 2 | 3 | 1 |
| JR. H. S. | 2 | 7 | 2 | 12 | 2 | 2 | 1 | 1 |
| GENERAL | 0 | 15 | 7 | 2 | 4 | 3 | 0 | 1 |

| | Total schools requiring | Total not requiring | Per cent not requiring | Per cent requiring 4 or more semester hours |
|---|---|---|---|---|
| RURAL | 52 | 11 | 17 | 19 |
| KINDERGARTEN—PRIMARY | 100 | 26 | 21 | 26 |
| INTERMEDIATE | 106 | 12 | 10 | 40 |
| GRAMMAR | 80 | 6 | 7 | 52 |
| JR. H. S. | 29 | 6 | 17 | 51 |
| GENERAL | 33 | 27 | 45 | 17 |

TABLE 4. Requirements in Arithmetic in Two-Year Courses

Table 4 gives the requirements in arithmetic in two-year courses. Using the second column as an example, Table 4 is read as follows: 20 schools require from 2 to 2.9 semester hours in arithmetic for rural school teachers; 45 schools require that amount of arithmetic for kindergarten-primary teachers; 31, for intermediate teachers; 27, for grammar grade teachers; 7, for junior high school teachers; 15, in a general course for teachers of all grades.

The column headed "Total schools requiring" gives the total number of schools requiring arithmetic in each of the different kinds of courses; for example, 52 schools offering courses for rural school teachers require arithmetic in those courses. The next column to the right gives the number of schools offering these different kinds of curriculums, but not requiring arithmetic in them; for example, 11 schools offering a curriculum for rural school teachers do not require arithmetic in it. The second column from the right gives the per cents of the two-year curriculums in which arithmetic is not required; for example, arithmetic is not required in 17% of the rural school curriculums, and in 45% of the general curriculums. Arithmetic is not required in 18% of all two-year curriculums offered, that is about one in six. It should be noticed that these requirements include all instruction in arithmetic, both in methods and in subject matter, that could be discovered in the printed curriculums of the 187 teacher-training institutions. The right hand column gives the per cents of two-year curriculums requiring 4 or more semester hours of arithmetic. Thirty-four per cent of all two-year curriculums require 4 or more hours of arithmetic. Before commenting upon the offerings and requirements in arithmetic shown in these tables, I wish to raise the question as to what training in arithmetic is needed by students in teachers colleges and normal schools, who are preparing to teach in the elementary school, and to give some evidence to answer that question.

## HOW MUCH ARITHMETIC DO COLLEGE FRESHMEN KNOW?

### THE ILLINOIS TEST

A test in arithmetic was given to 2,097 freshmen entering the five state teachers colleges of Illinois in the fall of 1927. The test consisted of 20 questions, 6 of which were two-step problems, 3 were one-step problems, and 11 were exercises in the fundamental operations with integers and common and decimal fractions.

The time given for the test was 45 minutes. At Charleston most of the students taking the test had time to complete it as far as they were able, and had time to reread and check their papers. Previous experiment had shown that the results from allowing the same or different weights to the questions affected the median and average only slightly. Hence for convenience in scoring, each correct answer was scored 5. In four of the colleges each question was scored right or wrong, questions not attempted being scored zero. In one college attempts and rights were scored. The questions are given below, and opposite each question is given the per cent of the 2,097 students failing to get the correct answer.

## SELECTIVE TEST ON FUNDAMENTALS OF ARITHMETIC

Arranged by C. N. Mills

Problems should be worked in sequence, and all work must be shown and neatly arranged. Time allowed, 45 minutes.

1. Add

   645.43
   784.05
   369.79
   858.88
   106.34
   966.97
   807.59 — 30%

2. Multiply

   86089
   9067 — 32%

3. At $8.25 a ton what is the cost of a load of coal weighing 6800 lb.? (1 ton = 2000 lb.) — 31%

4. Subtract

   458038
   288409 — 8%

5. Divide .35 | 175 — 46%

6. A man completed 5/8 of a piece of work in 7½ days. How long should it require him to complete the remaining 3/8 of the work? — 58%

7. Add the fractions 2/3 3/5 7/12 — 24%

8. In one year a cow gave 8600 lb. of milk, .04 of which was butter fat. At 48 cents a pound what was the value of the butter fat? — 39%

9. Multiply 24⅝ x 32¾ — 58%

10. The product of two numbers is 12¼, and one of the numbers is 3¾. What is the other? — 55%

11. James had 25¼ bu. of potatoes and sold 8 2/3 bu. How many bushels had he left? 35%

12. Divide 83 | 116781 19%

13. Add 14 7/8 40 5/6 32 2/3 32%

14. Change 7/16 to a decimal of four places. 44%

15. A man bought a house for $6350. After spending $236.60 for repairs, he sold the house for $8200. What did he gain? 27%

16. Material is bought for eight curtains each requiring 2¾ yards. What is the cost at 96 cents per yard? 27%

17. Divide 5/36 by 6 37%

18. A baker uses ⅝ lb. of flour to each loaf of bread. How many whole loaves can be made from a barrel of flour (196 lb.)? 57%

19. Divide 2.7 | .023328 42%

20. The list price of an article was reduced from $1.89 to $1.26. The reduction was what fractional part of the list price? 52%

The average score on the 20 questions in the five colleges was 60 on the basis of 100.

I have analyzed the errors in certain of the exercises in 270 papers. In .35 | 175 the quotient was found to be 50 by 2 students, 5 by 46, .5 by 18, and .05 by 41. Thirty per cent of the 270 failed to obtain the correct answer.

Of 139 failures on question 18, 64 multiplied by ⅝, 21 made an error in division, 12 in reasoning, 7 in multiplication, and 29 did not attempt or left incomplete.

The average for 270 students at Charleston was 61.2. The average for the third who had studied arithmetic since leaving the eighth grade, presumably for a semester in high school, was only 3.5 more, 64.7.

The evidence is conclusive that these students cannot be trusted to obtain correct answers and to check their results when performing the fundamental operations with integers. About three out of 10 failed in adding seven numbers of five figures each and in multiplying a five-place number by a four-place number; about 1 in 10 failed in subtracting one six-place number from another; about 1 in 5 failed in dividing a six-place number by a two-place number when the quotient was an integer. The domains of common and decimal fractions are largely unknown. About 3 out of 10 failed in adding 2/3, 3/5, and 7/12; 2 out of 5 failed in dividing 5/36 by 6; 5 out of 9 failed in dividing 175 by .35, and in reducing 7/16 to a decimal fraction. These seem sufficiently simple exercises. I am not surprised that these students made mistakes. That is to be expected. But that they should leave these errors without checking by some simple method proves that they need to be taught the subject matter of arithmetic before beginning to teach it.

I have mentioned that in finding how many loaves of bread can be made from 196 lb. of flour, allowing ⅝ of a pound to a loaf, 3 out of 7 students multiplied by ⅝. That is indicative of the kind of errors that are made in thinking about the simplest numerical relations as soon as common or decimal fractions are introduced into problems. No one except the teacher of arithmetic knows how many high school graduates cannot see that how many loaves, ⅝ of a pound to a loaf, is the same kind of question as how many loaves, 2 pounds to a loaf. How can problem solving be taught without seeing that?

Other studies have been made of the amount of arithmetic at the command of high school graduates and college students.

### SCHORLING AND CLARK'S TESTS

Schorling and Clark measured the ability of 3,545 children of grades from 5 to 12 in 100 simple tasks in computation. Of these, 215 were pupils in the twelfth grade. These are illustrative of the results. Of the pupils in the twelfth grade 29% could not find 25% of 80; 81% could not find 2.1% of 60; 12% could not add 7/8 and 3/16; 32% could not divide ⅜ by 4; 51% could not find the answer to: 6 is what per cent of 60?

Schorling and Clark say: "We find little increase in ability in computation after the eighth grade."

## COMPARISON OF COLLEGE FRESHMEN AND EIGHTH-GRADE PUPILS

Can college freshmen meet the requirements set up in standard tests for eighth-grade pupils?

R. L. Morton gave the Courtis Arithmetic Tests, Series B, to 104 women college freshmen at Ohio University. The results show that with Courtis' Standards as a basis, the average attainments of these students were about that of the 4th grade in addition, 5th in subtraction, between 4th and 5th in multiplication, and 6th in division. Of the 104 only one reached the Courtis eighth standards of speed and accuracy in the four fundamental operations with integers.

J. A. Drushel repeated Morton's test on 100 students at Harris Teachers College in St. Louis and found that 6 of these reached or surpassed the eighth-grade standards.

The Stanford Achievement Test, Form B, was given in the fall of 1927 to freshmen entering the Detroit Teachers College. The median score was 243. A state wide survey of the village and rural schools of the State of New York in April, 1926, gave approximately the same result as the average of the median scores made by eighth-grade pupils.

## DR. J. A. DRUSHEL'S STUDY

The most significant study of the abilities in arithmetic of prospective teachers that I know about is the doctor's thesis of Dr. J. Andrew Drushel of New York University on "Arithmetic Knowledges and Skills of Prospective Teachers." This study began with the entering class of the Harris Teachers College of St. Louis in September, 1907, and concluded with the entering class in 1924.

Dr. Drushel gave reasoning tests in arithmetic to 45 entering classes. These tests were made from problems selected from the 6th, 7th, and 8th grade books of the Southworth-Stone three-book series then in use in the St. Louis public schools. The problems were selected to cover as many phases as possible of the work of those years. Accuracy and skill in computing were measured by the scores on problems. During the latter part of the second period, the Courtis Research Tests, Series B, were used with 215 students, and the Cleveland Survey Tests with 141 students.

Each reasoning test was composed of 10 problems. The tests were given to freshmen in the early part of the first week of the first semester, in two fifty-minute periods, 5 problems in each period.

The students were from the upper two-thirds of their graduating classes in high school. They were solving problems from the books to be taught in the sixth, seventh, and eighth grades of the schools in which they were preparing to teach.

Of 11,293 problem opportunities there were 34.2% of correct answers. Certain types of errors that occur many times made it very clear that it is folly to expect these students to succeed in teaching arithmetic without giving them an extended course in the subject matter of arithmetic.

Over 30% of a group of 244 who needed the fact did not know the number of feet in a rod.

Of a group of 402 who attempted to get the hypotenuse of a right triangle when given the two legs, one-sixth of them either took the sum of the legs, the difference of the legs, the square root of the sum, or one-half the sum.

We are told that these students failed in the solution of problems for the same reasons that Osburn gives for the failures of elementary school children:

1. Partly because they relied upon memory of formal rules.
2. Partly because they read incorrectly.
3. Chiefly because they lacked proper methods of attack.

The solutions show generally inadequate training in number sense, in observing whether a result is reasonable, and in checking it. Dr. Drushel says:

"The majority of individuals in this study show an astonishing immaturity in ability to apply principles and processes in the solution of problems more difficult than the one-step type."

"About 15% (of 1,220 individuals whose papers were scored by the first method) show evidence of sufficient skill in problem solving to justify them in taking methods courses in the teaching of arithmetic without further study of content in connection with methods courses. About 10% of the group should have been eliminated at entrance."

In his conclusions regarding skill in computation Dr. Drushel says: "About 6% of the group reach or surpass the Courtis eighth grade rate-accuracy standards in each of the four fundamental operations with integers. The medians of these groups in the Cleveland Survey Tests in like and unlike fractions were below the medians of the St. Louis eighth grades."

It is not to be expected that college freshmen can compute as well as eighth grade pupils who are in practice. But what about the next eighth grade that has for a teacher one of these freshmen who has had no more instruction in arithmetic?

Dr. Drushel recommends that teacher-training institutions should give the necessary instruction *in the subject matter of arithmetic along with methods.* "It is still true that through content we must get method and that special method should not be superimposed upon content." This is a recommendation that needs serious attention in teachers colleges that require no arithmetic and in those that require and offer only courses in methods.

### THE NEED FOR INSTRUCTION IN ARITHMETIC

I have presented two sets of facts. One shows that most high school graduates have neither the skill in computation, mastery of facts, nor ability to solve problems to fit them to teach arithmetic. The other shows the amount of arithmetic offered and required in teacher-training institutions. It appears that a few institutions training teachers for the elementary schools offer no instruction in the subject matter or methods of teaching arithmetic; about one-sixth offer only methods courses; and about one-sixth of the two-year curriculums offered for elementary teachers require no work in subject matter or methods of teaching. This means that a large number of teachers may receive a two-year diploma without having any arithmetic beyond that taken in the eighth grade. A still larger number may, and no doubt do, graduate with no arithmetic, except a short course in methods of teaching.

About five-sixths of the curriculums for elementary teachers require two or more semester hours in arithmetic. Probably most of this instruction is in methods. What is a reasonable requirement? Four semester hours is a minimum. Table 4 shows the per cents of the curriculums in which four or more semester hours are required. About one-third of all curriculums (34%) require four or more semester hours of arithmetic. Dr. Drushel is clearly within the truth in saying that *only about one in six of our freshmen knows enough at entrance to profit by courses in methods.* It is folly to tell students how to teach decimal fractions and percentage who cannot divide 175 by .35 or find 2.1% of 180. It is far better to teach freshmen classes division of fractions well, leaving methods of teaching children to the training school, than to spend time talking about methods of presenting, motivating, and organizing subject matter that the students do not know. But a better method is to teach division of fractions to the college class by methods that they may use in the elementary school. This instruction, reinforced by practice teaching, should furnish a good preparation for teaching in the elementary school.

I know one Teachers College in which arithmetic has declined because the courses in arithmetic do not receive credit at the State University. Here is an example, that is bad, of abandoning the purpose of the Teachers College to meet the requirements of an outside standardizing agency. Arithmetic is a professional subject that is necessary for the best preparation of the elementary school teacher. It should not be dropped, and it should not be superseded by a course in more advanced mathematics. We can as little afford to substitute college algebra or calculus for arithmetic in the preparation of the elementary school teacher as can the engineering school afford to replace calculus by methods in arithmetic.

Is arithmetic a normal school subject but not a teachers college subject? Does a school preparing elementary teachers change its job when it changes its name? The schools in which arithmetic is approaching a vanishing point are nearly all in the part of the country where the name teachers college has recently become popular. Is it feared that the new academic dignity will suffer by offering a course in an elementary school subject? It should suffer if this course is eighth-grade arithmetic. It need not be. It should be a college course. There is an abundance of material. Professor C. B. Upton's articles in volume 27 of the Teachers College Record give an excellent exposition of it. For a good many years I have been teaching mathematics from arithmetic to calculus. I think that no course that I give is more valuable for its general culture, for its use in practical affairs, and none requires keener insight into quantitative relations than the course in arithmetic.

I began by referring to the progress made in the teaching of arithmetic in the last twenty years. I have not forgotten that. Neither am I overlooking the excellent preparation for the teaching of arithmetic given in many teacher-training institutions. But the teaching of arithmetic cannot make the progress it should in the next two decades if it is neglected in a large number of teachers colleges and normal schools. Progress must be stimulated in the main by them. Arithmetic has suffered by being called a tool subject. From which it is concluded that its mastery, like the use of a monkey wrench, depends only upon memory and practice. Fundamentally arithmetic is a tool in thinking, in determining relations of how many and how big. As such, it is immensely more important than reading and writing, convenient as they are. A great many people have gone a long way without being able to read or write. No one ever went any distance at all without being able to answer some questions of how many and how big. Professor Keyser is quoted as giving this idea this happy expression: "One may be a living being and not be able to count and measure, but one can not be a human being without being able to count and measure." Arithmetic, properly taught, belongs to the humanities after all.